Life-Cycle Wage Growth and Firm Productivity in Developed and Developing Economies

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Abstract

Life-cycle wage growth varies significantly across countries. We examine the role of the local distribution of firm productivity in shaping life-cycle wages by introducing a random search model that disentangles the effects of firm productivity distribution, on-the-job learning, and labor market frictions. Estimates of the model for Brazil, Colombia, and the United States suggest that the shape and scale of the firm-type distribution are key factors in explaining life-cycle wage growth. Counterfactual simulations suggest that equating the firm-type distribution in Brazil and Colombia to that of the United States would substantially reduce the cross-country gap in workers' life-cycle wage trajectories.

Keywords: Labor markets, Income profiles, Developing countries
JEL codes: J24, J31, J63, J64, E24, O11, O12, O15.

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1 Introduction

Life-cycle wage growth varies significantly across countries, with more developed economies experiencing much steeper wage growth trajectories (Lagakos et al., 2018). Differences in on-the-job learning and job mobility have been shown to contribute to these cross-country disparities in labor market outcomes, shaping workers' life-cycle wage profiles.[1] However, the steepness of these wage profiles also depends on the availability of better paying jobs within the labor market. Less developed economies often face a shortage of productive firms (Hsieh and Klenow, 2009; Eslava et al., 2022), limiting local workers' opportunities to increase their wages. This scarcity can constrain wage growth by restricting workers' access to higher-paying jobs. In fact, job-to-job mobility has been found to contribute substantially to earnings growth (Topel and Ward, 1992), suggesting that the availability of better paid jobs plays an important role in explaining cross-country differences in life-cycle wage growth.

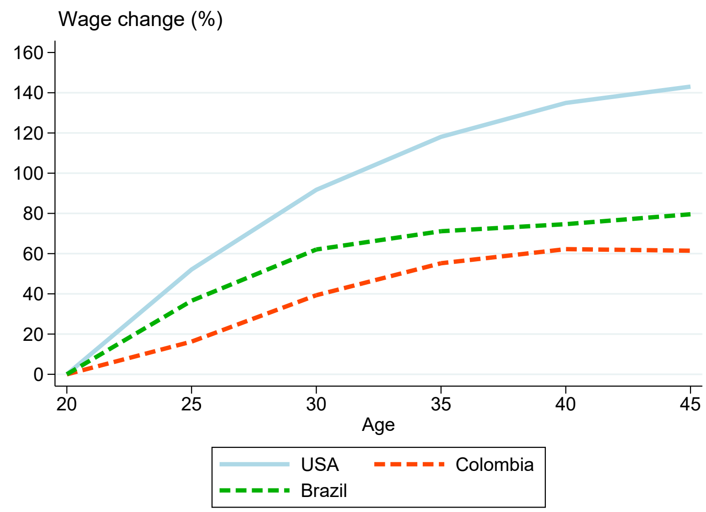
In this paper, we examine the role of the local distribution of firm productivity in explaining cross-country variations in life-cycle wage growth in the United States, Brazil and Colombia. To explore this relationship, we introduce a theoretical labor search framework that allows to disentangle the effects of firm productivity distribution, on-the-job learning, and labor market frictions in shaping wage profiles, building on Burdett and Mortensen (1998) and Bagger et al. (2014). Guided by the theory, we present empirical evidence showing a direct relationship between the firm type distribution and life-cycle wage growth. We then estimate the model using the simulated method of moments and find that the shape and scale of the firm-type distribution differ significantly across countries, with Brazil and Colombia characterized by a greater concentration of low-quality jobs and a relative scarcity of high-paying ones compared to the United States. Counterfactual simulations reveal that the firm-type distribution is a key factor explaining the gap in life-cycle wage growth between Brazil and Colombia relative to the United States.

We particularly focus on Colombia and Brazil, as recent research has highlighted the predominance of small, older establishments in these countries, which starkly contrasts with the firm-type distribution in the United States (Eslava et al., 2022, 2021). These contrasting environments may have significant implications for workers' life-cycle wage profiles, making them an important context for study. This contrast is illustrated in

[1] For the role of wage differences, see Lagakos et al. (2018). For differences in on-the-job training, see Ma et al. (2024) and Jedwab et al. (2023). For differences in job mobility, see Donovan et al. (2023) and Engbom (2022).

Figure 1, which plots life-cycle wage profiles for the three countries. Wage profiles are calculated as the average wage levels across different age groups, relative to workers aged 20 to 24.[2] Consistent with prior findings (Lagakos et al., 2018), the figure highlights a more pronounced life-cycle wage profile in the United States compared to Brazil and Colombia.

Figure 1: Age Profile of Wages



Notes: Wage change corresponds to the growth rate of average monthly wages for workers in each 5-year age bins relative to the average wages of workers between the ages of 20 and 24. Data for the USA comes from PSID, data for Brazil from PNADC, and data for Colombia from the social security records. Figure A.1 in the Appendix shows the same figure for additional data sets.

Our model provides a framework to examine the interplay between firm productivity, labor market frictions, and on-the-job learning. We show that on-the-job learning and labor market frictions have indirect equilibrium effects on the distribution of available jobs, which can reduce the likelihood of accessing high-paying positions. Intuitively, a firm productivity distribution with a longer right tail facilitates workers' advancement up the job ladder, increasing access to higher-paying jobs. However, beyond this direct effect, the model reveals two critical interactions between the firm-type distribution, on-the-job learning, and labor market frictions. First, higher rates of on-the-job learning lower workers' reservation wages, thereby extending the lower end of the job ladder. Additionally, higher on-the-job learning flattens the firm productivity-wage schedule. Second, higher job-finding

[2]Figure 1 employs data from the PSID for the United States, from the PNADC for Brazil, and from the social security records for Colombia (also known as the "Planilla Integrada de Liquidación de Aportes" (PILA)). Very similar patterns are observed when using alternative data sources and for the experience-income profile, as shown in Appendix Figures A.1 and A.2, respectively.

rates for employed workers, compared to the unemployed, further reduce reservation wages, reinforcing the extension of the lower tail. These dynamics underscore the crucial role that firm productivity heterogeneity plays in shaping lifetime earnings trajectories, both directly and through its interaction with labor market frictions and on-the-job learning.

We provide empirical evidence suggesting a strong correlation between life-cycle wage growth and the firm productivity distribution. Using cross-country, cross-regional, and individual-level variation, we identify a consistent correlation between life-cycle wage profiles and the firm productivity distribution. To proxy the productivity distribution, we use the share of workers employed at firms of different sizes —-a reasonable assumption based on our model and supported by previous empirical studies (Hsieh and Klenow, 2014; Eslava et al., 2023). Our findings indicate that workers in areas with a higher share of workers at larger firms experience steeper life-cycle wage growth. These results remain robust even when using worker-level data and controlling for time trends and time-invariant characteristics through individual fixed effects. Overall, our empirical analysis underscores a strong positive relationship between the distribution of firm productivity and workers' life-cycle wage growth.

We estimate the key parameters governing the firm-type distribution using the simulated method of moments by drawing on data moments that are commonly available in most longitudinal labor force surveys. Our estimation procedure enables a clear separation between the estimation of labor market parameters and that of the firm-type distribution, thereby strengthening the identification argument and the mapping between model parameters and data moments. We focus on several key metrics: the wage growth rate of job stayers, the life-cycle wage profile, unemployment and separation rates, and job-finding rates for both employed and unemployed individuals and show their relation to parameters in our model. Our model successfully captures cross-country differences in life-cycle wage growth, particularly the variations in the steepness of wage growth between the United States, Brazil, and Colombia.

Three key results emerge from the model's estimation. First, using the United States as a benchmark, we find that the distribution of firm productivity in Brazil and Colombia is heavily concentrated in low-quality jobs, with a scarcity of highly productive firms. This finding is consistent with Eslava et al. (2022), who also finds that smaller, likely unproductive firms account for a larger share of employment in Colombia compared to the United States. Second, learning on the job is slightly higher in the United States than in Brazil and Colombia, which supports the expectation that on-the-job training plays a significant role in driving income profile differences (Ma et al., 2024). Third, job-finding rates for the unemployed are lower in Brazil and Colombia compared to the United States, while job-finding rates for the employed are higher in these countries. This suggests that the difference in job-finding rates between the employed and unemployed varies by country income level. We validate this result using supplemental cross-country data, which provides further support for our finding. This insight has important implications: in more developed countries, where it is easier to find a job while unemployed, workers tend to be more selective in accepting offers. In contrast, in less developed countries, where it is easier to find a job while employed, workers are more likely to accept lower-quality jobs to climb the job ladder. As a result, the lower end of the job ladder is longer in developing countries, with a greater number of low-quality jobs available.

In a counterfactual analysis, we find that aligning the shape and scale of the firm productivity distribution in Brazil and Colombia with the United States' level results in the most substantial positive impact on life-cycle wage growth. This finding aligns with previous evidence comparing firm productivity across countries at different levels of development (Hsieh and Klenow, 2009). Specifically, matching the firm productivity distribution of Brazil and Colombia to the United States level would substantially reduce the gap in the steepness of the life-cycle wage growth trajectory between these countries. Additionally, aligning the level of on-the-job learning has significant effects for Colombia, increasing wage growth from 52 percent to 70 percent of the United States' value, but no effect for Brazil.

Our results indicate that life-cycle wage growth disparities are not solely driven by learning on the job and the likelihood of job mobility, but most strongly by the firm productivity distribution. Although our framework adopts a specific structure for wage bargaining and wage growth, our estimation results can be interpreted more broadly. In particular, within a wide class of labor search models that allow for separate identification of labor market frictions and firm heterogeneity, we discuss that our findings can be interpreted as a likely lower bound on the role of the firm productivity distribution in explaining cross-country differences in wage growth.

This paper builds on three key areas of literature. First, it contributes to the body of work linking life-cycle wage growth to a country's level of development. These studies highlight variations in income growth and suggest that differences in human capital accumulation are key to explaining the disparities observed across countries. For example, Lagakos et al. (2018) document that wage growth in developed economies is, on average, twice as steep as in developing economies. Fang and Qiu (2022) observe a similar pattern when comparing China with the United States. Jedwab et al. (2023) find analogous trends across 145 countries, arguing that workers in developed economies accumulate more human capital on the job. Ma et al. (2024) further suggest that more on-the-job training in developed economies partially explains these wage growth disparities. Guner et al. (2018) show that life-cycle earnings growth for managers relative to non-managers increases with economic development, proposing that distortions disincentivize learning. Engbom (2022) finds that wage growth is greater in countries with higher job-to-job mobility, explaining 50 percent of cross-country life-cycle wage growth differences. Additionally, Donovan et al. (2023) shows that job-finding and employment-exit rates are negatively correlated with development, emphasizing the role of labor market frictions in explaining wage growth differences. We contribute to this literature by emphasizing the role of the firm productivity distribution as a fundamental determinant of life-cycle wage growth.

Second, this paper contributes to the literature on random search labor models with the objective of explaining income profiles both within and across countries. While there exists a substantial body of literature explaining cross-country differences in mobility rates and labor market outcomes (Jolivet et al., 2006), as well as explaining wage growth rates on-the-job (Rubinstein and Weiss, 2006; Barlevy, 2008; Yamaguchi, 2010; Gregory, 2020), only a few studies focus on decomposing wage growth patterns. Notably, Bagger et al. (2014) and Menzio et al. (2012) decompose wage growth into human capital accumulation and job search using two distinct labor search frameworks. Building upon the former, Ozkan et al. (2023) show that human capital growth patterns are important determinants of income differences within the United States economy. Similarly, Burdett et al. (2011) suggest a model related to Burdett and Mortensen (1998) incorporating human capital accumulation but without firm heterogeneity and with production complementarity. However, unlike us, their aim is not to explain cross-country differences in wage growth rates. In contrast, we focus on the role of the firm productivity distribution in explaining the steepness of life cycle-wage trajectories across countries.

Third, we contribute to the literature that highlights the role of the distribution of firm productivity for development. This body of literature shows that the growth rate of manufacturing plants during the life cycle is lower in less-developed countries and that the variance of firm productivity within narrowly defined industries is higher in less developed countries (Hsieh and Klenow, 2009, 2014; Hsieh and Olken, 2014; Poschke, 2018). Specifically for Latin America, Eslava et al. (2021) and Eslava et al. (2023) address this topic by showing that the firm size distribution in the region exhibits a predominance of small businesses, and this is partly explained by a slower exit rate of smaller firms (Eslava et al., 2022). Our paper suggests that a compressed firm productivity distribution with less outstanding firms can contribute to the lower slope of the life cycle wage growth of workers in developing countries.

The paper proceeds as follows. Section 2 presents the theoretical framework. Section 3 summarizes our data sources, and section 4 provides the empirical motivation. Section 5 describes the parametrization and calibration procedures used to identify the model. Section 6 displays the results by presenting the model estimates and the counterfactual analysis. Finally, Section 7 draws the conclusion.

2 Framework

We present a parsimonious random search model that allows to quantitatively assess the relevance of the firm-productivity distribution on life-cycle wage profiles. The model builds on Burdett and Mortensen (1998) but incorporates worker heterogeneity, learning, and wage piece-rates.

2.1 Setting

The economy is populated by a continuum of workers of size M and a mass of firms of size L. Time is discrete, and workers and firms discount the future at a common rate $\tilde{\beta}$. Each period a measure of workers ν dies and an equivalent measure of worker is reborn.[3] Denote with $\beta = (1-\nu)\tilde{\beta}$ the aggregate effective discount rate. Workers and firms have heterogeneous productivitiy, indexed by p for firms and h for workers. Workers can be employed or unemployed and a measure of workers $\tilde{U}$ is unemployed. Employed and unemployed workers have a chance of meeting a new firm at probability λ_1 and λ_0, respectively, and when employed, they face an exogenous probability of match dissolution δ. Workers are endowed with an initial level of skills $h_0 \geq 0$ that grows, when employed,

[3] Our model abstracts from life-cycle considerations, as job acceptance decisions in our framework are not influenced by life-cycle motives, as in Bagger et al. (2014). Incorporating endogenous learning decisions, or age variation in learning, however, could introduce such life-cycle elements, which are outside of the scope of our model.

at rate μ. The skills of employed workers then follow the law of motion:

$$h' = h + \mu.$$

Workers are entitled to receive unemployment benefits of the form:

$$b(h) = b_0 + h.$$

Firm productivity is distributed according to the distribution $\Gamma(p) \in [\underline{p}, \infty]$. We denote its anti-cumulative distribution function as $\bar{\Gamma}(p) = 1 - \Gamma(p)$ and its probability density function as $\gamma(p)$.

Output y of the worker-firm match is additive in the two productivity terms of the worker-firm match:

$$y = p + h.$$

Firms post wage components, $w_0(p)$, as the sum of wage piece-rates, $r(p)$, and productivity p of the firm, such that total wages $w(p, h)$ follow:

$$w(p, h) = r(p) + p + h = w_0(p) + h := w(w_0, h) \tag{1}$$

The endogenous distribution of wages is denoted $F(w)$, such that $w \in [\underline{w},\overline{w}]$ with anti-cumulative distribution function $\bar{F}(w) = 1 - F(w)$.

2.2 Value Functions and Firm Problem

Let $U(h)$ denote the value of an unemployed worker with productivity h, and let $W(w_0, h)$ denote the value of an employed worker with skills h at a firm offering w_0. The utility of an unemployed worker is composed of the flow value of unemployment $b(h)$ and the option value of firm matching. When matching with a firm at rate λ_0, the worker is promised utility $W(w_0(p), h)$ and moves to the new job if the value at the job exceeds the value in unemployment. We can hence write the worker's lifetime value in unemployment as follows:

$$U(h) = b(h) + \beta(1 - \lambda_0)U(h) + \beta\lambda_0 \int \max\{U(h), W(w_0, h)\} dF(w_0).$$

When employed at a firm offering wage component w_0, the worker receives wages $w(w_0, h)$ and faces the option value of learning and mobility to other jobs as well as the option value of unemployment due to displacement. Specifically, at exogenous probability δ, the worker is displaced from his current job and becomes unemployed, yielding the value $U(h)$. When the worker is not displaced, he learns, increasing his human capital to value h'. He has then the opportunity to meet another firm. At probability λ_1, the worker meets an outside firm and decides whether to move to the new firm or stay with the incumbent firm. The worker's value when employed is then:

$$
\begin{aligned}
W(w_0, h) &= w(w_0, h) + \beta\delta U(h) + \beta\lambda_1 \int \max\{W(w_0, h'), W(x, h')\} dF(x) \\
&+ \beta(1 - \delta - \lambda_1) W(w_0, h').
\end{aligned}
$$

Firms offer wage contracts $w_0(p)$ that maximize total firm profits. Total profits of the firm are obtained as the product of per-worker profits $y - w(p, h) = p - w_0$, which are constant within the firm due to the piece-rate policy, and firm-size $l(w_0)$. Formally, firms' profits are hence expressed as:

$$
\pi(p) = \max_{w_0} (p - w_0(p))\, l(w_0(p)).
$$

Firm size is a function of the job finding rate, λ_1, and the separation rate, δ, given some constant A such that:

$$
l(w_0(p)) = \frac{A}{\left[\delta + \lambda_1 \overline{F}\,(w_0(p))\right]^2}.
$$

where A is a constant.[4]. The contract distribution must in turn satisfy the equilibrium condition $\Gamma\,[p] = F(w_0(p))$.

2.3 Equilibrium

The model's equilibrium is governed by a reservation wage expression and a wage equation.

Reservation Wages As typical in this type of model, there exists a reservation wage component, $\theta^R = w^R(p, h) - h$, such that an unemployed worker will accept the current job offer as the value of the job is identical to the value of unemployment, that is $W(\theta^R, h) = U(h)$. We guess and verify an equilibrium in which the reservation wage component is identical for all workers and show its derivation in Appendix section B.2. We obtain the reservation wage component as:

[4]We present the derivation of firm size in the model in Appendix section B.1

$$\theta^R = \underbrace{b_0}_{\text{Unemployment Benefit}} \underbrace{- \beta(1-\delta)\frac{\mu}{1-\beta}}_{\text{Rosen Effect}} + \underbrace{\beta(\lambda_0 - \lambda_1)\int_{\theta^R}^{\overline{w}} W_x \overline{F}(x)dx}_{\text{Entry Effect}}, \tag{2}$$

which is composed of three parts: the unemployment benefit; a part due to learning on the job, denoted *Rosen Effect*; and a component due to differences in job finding rates when employed or unemployed, referred to as *Entry Effect*. Note that without learning on the job ($\mu = 0$) and without differences in job finding on and off the job ($\lambda_0 = \lambda_1$), the reservation wage component would just be equal to the unemployment benefit component b_0. In the presence of learning ($\mu > 0$), the reservation wage component is lower, all else equal. As workers do not want to forgo human capital increases due to learning on the job, they are willing to accept worse job matches in terms of the wage component. In other words, an economy with higher on-the-job learning, μ, has a longer left tail of the firm type distribution and hence a longer part of the job ladder with low quality jobs. This mechanism was first emphasized by Rosen (1972) and is present in our framework as well. We denote this mechanism as the *Rosen Effect*. Finally, a third term emerges due to the difference in job finding rates on and off the job. If it is easier to find a job when unemployed then when employed ($\lambda_0 > \lambda_1$), then the reservation wage component increases in the option value of on-the-job search, $\int_{\theta^R}^{\overline{w}} W_x \overline{F}(x)dx$. In other words, the worker becomes more selective knowing that it will be harder to find a new match when employed than when unemployed. Reversely, if it is easier to find a new job when employed than when unemployed ($\lambda_1 > \lambda_0$), workers are willing to accept a lower wage component to get access to a better job search prospect. We denote this latter effect the *Entry Effect*.

Wage Equation: As shown in appendix section B.3, the firm problem implies the wage equilibrium condition:

$$\frac{p - w_0(p)}{\left[\delta + \lambda_1 \overline{\Gamma}\left(w_0(p)\right)\right]^2} - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda_1 \overline{\Gamma}\left(x\right)\right]^2} dx. \tag{3}$$

Note that no firm will offer a wage component below the reservation wage component, such that profits at the lower productivity bound are equal to zero ($\pi(\underline{p}) = 0$). This further pins down the effective lower level of productive firms in the economy $\underline{p} = \theta^R$. We can rewrite Equation 3 to obtain the wage equation as:

$$w(p,h) = h + p - \left(1 + k_1\overline{\Gamma}(p)\right)^2 \int_{\underline{p}}^{p} \frac{1}{\left(1 + k_1\overline{\Gamma}(x)\right)^2} dx,$$

with $k_1 = \lambda_1/\delta$ and $w_0(p) = w(p,h) - h$.

The wage equation is composed of a worker specific component, h, and a firm specific component, $p - \left(1 + k_1\overline{\Gamma}(p)\right)^2 \int_{\underline{p}}^{p} \frac{1}{\left(1+k_1\overline{\Gamma}(x)\right)^2} dx$. The latter is also composed of two terms. The first is increasing in firm productivity and represents the direct contribution of productivity to wages. The second is decreasing in firm productivity and represents the rent share component. The literature has discussed that more productive firms have a lower rent share component due to fewer competition from other firms (Bontemps et al., 2000). The extend of this effect depends on the firm-productivity distribution, as given by $\Gamma(p)$, and labor market frictions, represented by the parameter k_1.[5]

2.4 Comparative Statics

The model helps us understand the complex interrelationship between learning rates, labor market frictions, and the firm-productivity distribution in shaping the life-cycle wage profile of workers. We can distinguish between direct and indirect effects of these factors on wage profiles. The direct effects emanate from the insight that all else equal, less labor market frictions (higher λ_1, λ_0, lower δ) will increase the life-cycle wage growth rate in a simulated labor market history. Similarly, a higher rate of on-the-job growth (higher μ) will increase the wage at a given age. The indirect effects derive from the effect of model parameters on the wage-productivity schedule ($\partial w_0/\partial p$), which can be signed using the equilibrium conditions of the model.[6] Table 1 summarizes these indirect effects .

To fully analyze the effect of the characteristics of the firm type distribution, we proceed by specifying its functional form as a Pareto distribution with scale parameter $\underline{p}$ and shape parameter α, such that $\Gamma(p) = 1 - \left(\frac{\underline{p}}{p}\right)^{\alpha}$, following Ozkan et al. (2023) and Hubmer (2018). A larger scale parameter, $\underline{p}$, shifts the lower end of the firm type distribution upwards. In

[5] Formally, the direct contribution of productivity to wages increases 1:1 with productivity, whereas the rent share component increases in firm productivity at rate

$$\frac{\partial w(p,h) - h - p}{\partial p} = -\left(1 - 2(1 + k_1\overline{\Gamma}(p))k_1\gamma(p) \int_{\underline{p}}^{p} \frac{1}{(1 + k_1\overline{\Gamma}(x))^2} dx\right).$$

[6] The derivations are detailed in Appendix Section B.4.

contrast, a larger shape parameter, α, leads to a smaller mass of firms at the right tail of the distribution.[7] In other words, a larger $\underline{p}$ implies a higher minimum firm productivity, while a larger α implies lower predominance of high-productivity firms.

Table 1: Effects on Wage-Productivity Schedule

Labor market frictions	Job Finding Rate (Employed)	λ_1	$\frac{\partial^2 w}{\partial p \partial \lambda_1} > 0$
	Job Finding Rate (Unemployed)	λ_0	$\frac{\partial^2 w}{\partial p \partial \lambda_0} = 0$
	Separation Rate	δ	$\frac{\partial^2 w}{\partial p \partial \delta} < 0$
Learning on the job	Learning Rate	μ	$\frac{\partial^2 w}{\partial p \partial \mu} < 0$
Firm productivity	Scale (Left-tail)	$\underline{p}$	$\frac{\partial^2 w}{\partial p \partial \underline{p}} > 0$
	Shape (Right-tail)	α	$\frac{\partial^2 w}{\partial p \partial \alpha} >< 0$

Indirectly, the model suggests that the steepness of the wage-productivity schedule increases with higher job finding rate λ_1 ($\frac{\partial^2 w}{\partial p \partial \lambda_1} > 0$), lower exogenous separation rate δ ($\frac{\partial^2 w}{\partial p \partial \delta} < 0$), and higher reservation productivity level θ^R ($\frac{\partial^2 w}{\partial p \partial \theta^R} > 0$). Higher on the job learning rates, μ, depress this lowest productivity boundary, such that there is a negative relationship between learning on the job and the steepness of the wage-productivity schedule in this model ($\frac{\partial^2 w}{\partial p \partial \mu} < 0$). Hence, while higher rates of on-the-job learning increase the steepness of the life-cycle wage profile directly, they indirectly lower the possibility of wage growth through on-the-job search. Finally, a higher tail parameter of the firm type distribution (α) has a different effect depending on the level of firm productivity - with an increasing relationship for small values and a decreasing relationship for high values. This is intuitive as the wage-productivity schedule varies in a firm's labor market competition, which raises for low productive firms if the share of high productive firms falls. As a result, low productive firms need to increase their wage offer to workers.

These insights show that the firm productivity distribution affects wage growth through direct and indirect mechanisms which interact with other parameters in the model. Overall, these interactions raise the question of the importance of the firm type distribution holding these other factors fixed.

[7] Intuitively, extraordinary firm productivities at a high frequency require a low shape parameter α. Similarly, for $\alpha > 2$, the variance and mean of firm productivity decrease in α.

3 Data

3.1 Data Sources

To take the model to the data, we leverage both longitudinal and cross-sectional data regarding workers' mobility and wages. We use diverse data sources that extensively cover labor markets over time in the United States, Brazil, and Colombia, leveraging the unique strengths of each dataset to address various aspects of workers' labor market histories. These three countries vary in the steepness of their workers' life-cycle wage profiles, providing us with alternative scenarios that vary on their level of economic development.

For the United States, we employ the outgoing rotation group of the *Current Population Survey* (CPS) from 2003 to 2013.[8] Households in the CPS are initially interviewed and then followed for four consecutive months. After that, they are rotated out for the next eight months before being surveyed again for four consecutive months one year after the first interview.[9]

For Brazil, we analyze the *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC) data from 2012 to 2019. This nationally representative survey, initiated in 2012, is designed to monitor the dynamics of Brazilian labor markets. Workforce indicators are collected quarterly, and selected households are interviewed for five consecutive quarters.

For Colombia, we combine household surveys and social security records to provide a more comprehensive perspective on the labor market. The household surveys are derived from the *Gran Encuesta Integrada de Hogares* (GEIH), which is officially compiled on a monthly basis to track household dynamics in the country. While these data do not track workers longitudinally, they include a series of retrospective questions that allow for the calculation of job-mobility measures, including transitions into and out of unemployment. However, since these data are not longitudinal, they hinder the computation of individual wage growth. To address this limitation, we rely on social security records from the *Planilla Integrada de Liquidación de Aportes* (PILA), which covers the entire population of formal workers who held a formal job from 2009 to 2016.[10]

[8] These years are chosen to be comparable with Lagakos et al. (2018).

[9] Additionally, we use the *Panel Study of Income Dynamics* (PSID) from 1975 to 2013, and the *National Longitudinal Survey of Youth* (NLSY) from 2000 to 2019, to compute life-cycle wage profiles. These profiles are used to validate the measures computed in the CPS. The PSID, for instance, has been used in previous studies to compute life-cycle wage profiles (Lagakos et al., 2018).

[10] Formal workers are defined as those who contribute to health and pension, comprising approximately

3.2 Key Variables

Four groups of key variables are included in our analysis. First, we compute wage measures across all datasets. To ensure comparability, we harmonize these measures by focusing on monthly wages, as hourly wages cannot be consistently computed across all datasets. These wage measures are deflated and expressed in U.S. real Dollars to maintain comparability. Additionally, we winsorize wages at the first and 99th percentiles to handle outliers consistently.

Second, we compute measures of individual wage growth rates. For the United States and Brazil, we can directly calculate quarterly wage growth rates due to the longitudinal nature of the CPS and the PNADC. However, we cannot do the same for the Colombian household surveys, as they are not longitudinal data sets. We compute wage growth rates for Colombia using administrative social security records, expressed at a quarterly frequency. A limitation of this approach is that informal sector workers are not covered in these records, which restricts our analysis to formal sector employment. However, this restriction enables consistent harmonization across data sources, and we view it as a necessary feature of our empirical framework.

Third, we compute measures of job mobility. Across datasets, we define individuals working in the same job as *job stayers*, while those changing jobs are categorized as *job movers*. We restrict our analysis to individuals who are either employed or unemployed, excluding those out of the labor force. In the CPS, a *job stayer* is defined as someone who reports working for the same employer for four consecutive months. In contrast, *job movers* are those who change employers at any point during those four months. In the PNADC, a *job stayer* is someone who has been in the same job for at least one quarter and a *job mover* is its complement. Therefore, job mobility rates in the CPS and the PNADC are calculated quarterly. For Colombia, we use household surveys and follow the methodology outlined by Lasso (2013) to build measures of job mobility using retrospective data. Even though job mobility rates for Colombia are calculated annually, we transform them into quarterly rates to be comparable to the United States and Brazil.

Finally, we build measures of firm size for employed individuals. We can only build measures of firm size at the individual level for Brazil, since the PNADC is the only longitudinal dataset among the three countries with information about firm size. The

60 percent of jobs.

GEIH for Colombia includes this information but is not longitudinal, which prevents us from computing wage progression by firm size. Similarly, while the longitudinal panel of the CPS does not include information on firm size, its cross-sectional versions do. Therefore, we compute aggregated measures of firm size at the state level for the United States and Colombia.

3.3 Sample

Our analysis is based on workers observed between the ages of 20 and 60. Table 2 provides a summary of the main datasets used in the study. The samples across all three countries are predominantly male (approximately 60 percent), with the average age being 41 for the United States, and 38 for Brazil and Colombia. Additionally, wage growth for job movers tends to be higher than that for job stayers, consistent with the idea that wage growth is primarily driven by job-to-job transitions.

4 Empirical Motivation

Building on the model's insight, we leverage cross-country, cross-regional, and individual-level variation to provide three pieces of evidence that reveal a consistent correlation between life-cycle wage profiles and the firm productivity distribution. To capture the life-cycle wage profile, we compute the ratio of wages for workers aged 40-44 relative to those aged 20-24. Since the firm productivity distribution is not directly observable, we use the share of workers employed in firms with more than 10 and 50 employees as proxies for firm productivity characteristics. While the distribution of firm productivity and firm size are not identical, we show in Appendix Section B.5 that this proxy is supported by our model, aligning with prior empirical evidence (Hsieh and Klenow, 2014; Eslava et al., 2023).[11]

We first consider cross-country correlations, which are presented in Figure 2. We complement this analysis using the *European Union Statistics on Income and Living Conditions* (EU-SILC) data from 2005 to 2019.[12] We find consistent positive correlations between life-cycle wage growth and our measures of firm size, which we argue proxy for the distribution of firm productivity. These results complement the conventional view

[11]Specifically, we demonstrate that the share of workers in firms with a given productivity level increases with firm size.

[12]Data from EU-SILC cover an extensive number of European countries and is collected as a rotating yearly panel that surveys individuals over several periods. We use these data to create cross-country correlations in our measures of interest.

Table 2: Summary Statistics

	Obs. (1)	Mean (2)	SD. (3)	Median (4)	Min. (5)	Max. (6)
A) USA (CPS)						
Age	947,362	40.853	11.00	41.00	20.00	60.00
Female	947,362	0.464	0.50	0.00	0.00	1.00
Log(Monthly Wages)	826,668	7.928	0.70	7.95	5.69	9.35
1(Stayer)	947,362	0.928	0.26	1.00	0.00	1.00
Job-to-Unemployment	947,362	0.020	0.14	0.00	0.00	1.00
Job-to-job	947,362	0.052	0.22	0.00	0.00	1.00
Unemployment-to-job	51,130	0.456	0.50	0.00	0.00	1.00
Wage Growth for Stayers	202,015	0.019	0.49	0.01	-3.66	3.66
Wage Growth for Mover	8,962	0.039	0.54	0.02	-3.66	3.20
B) Brazil (PNADC)						
Age	4,497,588	37.967	10.73	37.00	20.00	60.00
Female	4,497,588	0.421	0.49	0.00	0.00	1.00
Log(Monthly Wages)	4,136,189	6.289	0.83	6.23	3.37	8.48
1(Stayer)	4,497,588	0.891	0.31	1.00	0.00	1.00
Job-to-Unemployment	4,497,588	0.033	0.18	0.00	0.00	1.00
Job-to-job	4,497,588	0.075	0.26	0.00	0.00	1.00
Unemployment-to-job	325,752	0.440	0.50	0.00	0.00	1.00
Wage Growth for Stayers	3,749,814	0.005	0.44	0.00	-5.11	5.11
Wage Growth for Mover	280,053	0.013	0.64	0.00	-4.62	4.09
C) Colombia						
Age	2,508,256	37.814	10.99	37.00	20.00	60.00
1(female)	2,508,256	0.415	0.49	0.00	0.00	1.00
log(Monthly Wages)	2,047,820	5.622	0.90	5.67	2.76	7.74
1(Stayer)	2,508,256	0.636	0.48	1.00	0.00	1.00
Job-to-Unemployment	2,508,256	0.080	0.27	0.00	0.00	1.00
Job-to-job	2,508,256	0.284	0.45	0.00	0.00	1.00
Unemployment-to-job	123,191	0.838	0.37	1.00	0.00	1.00
Wage Growth for Stayers*	31,999,316	0.023	0.36	0.01	-17.13	16.89
Wage Growth for Mover*	17,306,998	0.042	0.54	0.02	-18.76	17.06

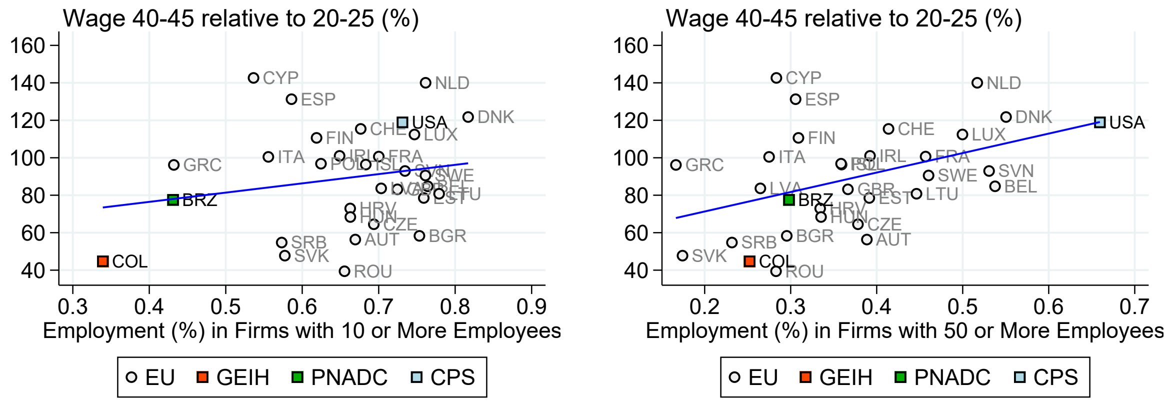
Notes: All wages are expressed in 2010 USD, and winsorized in the 1st and 99th percentile. Brazilian (PNADC) data have quarterly frequency, and CPS data are collected monthly for four consecutive months. Therefore, job mobility measures and wage growth rates in the CPS and PNADC are computed quarterly. Colombian measures are computed yearly. All samples are restricted to workers between the ages of 20 to 60. *Computed using the Colombian social security records.

that on-the-job learning and labor market frictions are the major drivers of cross-country differences in life-cycle wage profiles, although no direct causality is implied.

Next, we zoom in on our three countries of interest—Brazil, the United States, and Colombia—and compute cross-regional correlations within each country. Figure 3 presents state-level correlations within these countries. Between countries, we observe that life-cycle wage profiles are steeper in the United States, where employment is more concentrated in

Figure 2: Life-Cycle Wage Profiles and Firm Size Across Countries

(a) Firms with 10 or More Employees (b) Firms with 50 or More Employees



Notes: These figures combine data from EU-SILC (gray markers), CPS (light blue markers), PNADC (green markers), and GEIH (red markers). The y-axis corresponds to the ratio of monthly wages paid to workers between 40-44 years old, relative to those with 20-24 years of age. The x-axis in Figure 2a and Figure 2b corresponds to the share of employment in firms with more than 10 and 50 employees, respectively. Each point corresponds to a time-invariant value computed first by collapsing individual-level data at the country-year level, and then averaged across years. We exclude data from Portugal, Germany, and Norway from our analysis. Portuguese data differ in how incomes were recorded, Norwegian data recorded job transitions differently, and German data were only available for two periods with a very small sample size.

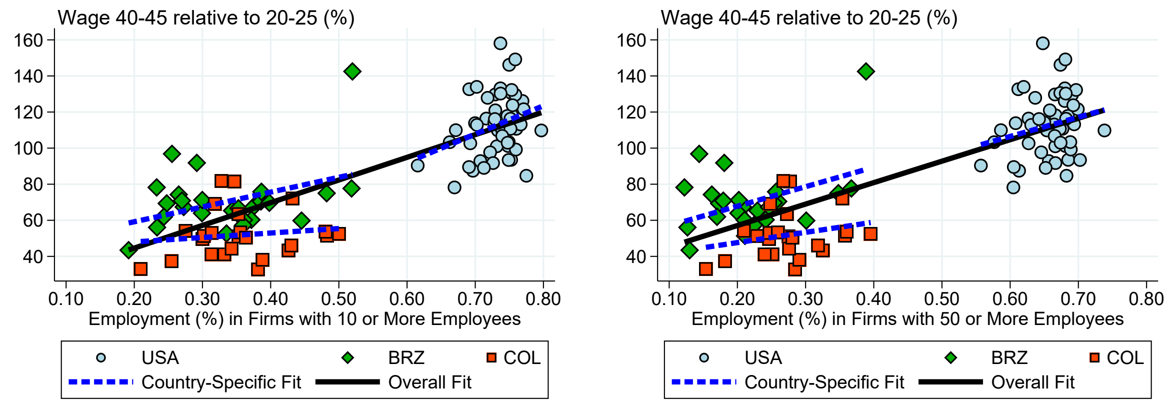
larger firms. States in Colombia and Brazil show a less steep gradient in life-cycle wage profiles and a noticeably smaller share of employment in larger firms. Within countries, we observe a similar positive correlation between life-cycle wage profiles and the share of employment in bigger firms for all three countries.

Cross-country and cross-regional correlations, however, may be susceptible to various biases, especially due to comparability issues arising from differences in sample composition –for instance, more skilled workers tend to be employed by larger firms, and vice versa. To address these concerns, we estimate individual-level Mincer regressions that control for time-invariant individual characteristics and time trends, along with the measures of firm size where the individual is employed. As explained in Section 3.2, we use worker-level panel data from Brazil (PNADC), which is the only longitudinal dataset that includes wage and firm size information. We control for individual-level characteristics to mitigate potential concerns related to sample selection. Formally, we estimate the following regression:

$$\ln w_{it} = \sum_{k \neq [0-5]} \beta_k 1(FS_{it} = k) + \gamma a_{it} + \delta a_{it}^2 + \mu_i + \mu_s + \mu_t + \varepsilon_{it}, \tag{4}$$

Figure 3: Life-Cycle Wage Profiles and Firm Size Within Countries

(a) Firms with 10 or More Employees (b) Firms with 50 or More Employees



Notes: These figures combine data from the CPS (light blue markers), PNADC (green markers), and GEIH (red markers). The y-axis corresponds to the ratio of monthly wages paid to workers between 40-44 years old, relative to those with 20-24 years of age. The x-axis in Figure 3a and Figure 3b corresponds to the share of employment in firms with more than 10 and 50 employees, respectively. Each point corresponds to a time-invariant value computed first by collapsing individual-level data at the country-state-year level, and then averaged across years.

where $\ln w_{it}$ corresponds to log wages for individual i in year t, a_{it} represents age, and μ_i, μ_s and μ_t correspond to individual, state, and year fixed effects, respectively. The variables of interest are represented by $1(FS_{it} = k)$, which are dummy variables equal to one if the individual i is employed in a firm of size k at period t. We include three size categories—$k \in \{6\text{-}10, 11\text{-}50, > 50\}$—and use firms with 1 to 5 employees as the omitted reference group. The inclusion of individual fixed effects helps isolate within-worker variation, thereby reducing concerns about sample composition across firms. Standard errors are clustered at the state level.

We present the estimation results in Table 3, which confirm a positive association between wages and firm size. Column (1) reports point estimates without controlling for individual characteristics, showing a monotonic increase in wages with firm size. This pattern may reflect worker sorting, where more skilled individuals tend to be employed by larger firms. Column (2) mildly addresses this concern by including individual-level controls. Although the estimates decrease slightly, the monotonic relationship remains. In column (3), we incorporate individual fixed effects to account for time-invariant worker characteristics. These within-individual comparisons mitigate concerns related to differences in sample composition across firm sizes. The estimates decline more substantially and the monotonicity persists, suggesting that the same individual earns higher wages when employed by larger firms—our proxy for firm productivity.

To more directly examine the role of the job ladder in shaping wages, we additionally split the sample and estimate the model separately for *job stayers* and *job movers*. Results are presented in columns (4) and (5) of Table 3. In both groups, wages increase monotonically with firm size, with a more pronounced gradient among job movers. This reinforces the conclusion that firm size plays a central role in determining wage outcomes. Overall, these results provide strong evidence that the distribution of firm types significantly influences wages and, ultimately, life-cycle wage growth.

Table 3: Individual Wages and Firm Size

	Overall			Job Stayers	Job Movers
	(1)	(2)	(3)	(4)	(5)
6-10 Emp.	0.245***	0.227***	0.085***	0.048***	0.171***
	(0.043)	(0.030)	(0.007)	(0.004)	(0.019)
11-50 Emp.	0.326***	0.278***	0.126***	0.074***	0.226***
	(0.051)	(0.035)	(0.011)	(0.006)	(0.029)
50 or More	0.422***	0.323***	0.147***	0.084***	0.271***
	(0.049)	(0.032)	(0.012)	(0.006)	(0.028)
Observations	4,571,895	4,571,895	4,019,253	2,286,272	81,835
State FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Controls		Yes	Yes	Yes	Yes
Individual FE			Yes	Yes	Yes

Notes: The table shows the results of estimating Equation 3 using log wages as outcome, and performed in the longitudinal module of the Brazilian PNADC. Excluded category corresponds to individuals working in firms with one to five employees. Individual controls include age, age-squared, a dummy for gender and years of education. Columns (3) to (5) include individual fixed effects, therefore they drop the dummy for female and years of education but still control for age and age-squared. Column (4) includes individuals who did not change job, whereas column (5) includes people who changed jobs. Estimations weighted by survey weights. *** p<0.01, ** p<0.05, * p<0.1.

5 Identification

We use the previously described datasets to estimate the parameters of the model. These parameters fall into three categories: i) firm productivity distribution ($\underline{p}, \alpha$); ii) labor market frictions ($\lambda_1, \lambda_0, \delta$); and iii) on-the-job learning (μ).[13] In our estimation procedure, we can separate the estimation of the labor market parameters and the parameters of the firm type distribution, which significantly facilitates estimation and improves transparency of the estimation procedure and identification. Such a procedure has recently been employed in Moscarini and Postel-Vinay (2023). We will hence start to show identification of the labor market parameters before moving on to the parameters governing the firm type distribution.

Labor Market Frictions: To estimate the labor market parameters, we leverage two identities. First, we employ the steady-state flow equation for unemployment. In steady state, the inflow of employed workers into unemployment through displacement at rate $(1-u)\delta$ and the outflow from unemployment through job finding at rate $\lambda_0 u$ holds a balance such that:

$$\frac{1-u}{u} = \frac{\lambda_0}{\delta} = k_0.$$

The job finding rate from unemployment, λ_0, can be directly estimated from the data using $\hat{\lambda}_0 = E[UE]$, which corresponds to the share of unemployed workers who find a job. Hence, we can deduce and estimate the exogenous displacement rate from $\hat{\delta} = \hat{\lambda}_0/\hat{k}_0$.

Second, we use the expected value of the separation rate, which is equivalent to the sum of the job-to-job transition rate (EE rate) and the exogenous displacement rate (δ), which can be recovered in closed form. Denote with $g(w_0) := g(w)$ the distribution of wage components across workers. In Appendix section B.6, we show the mapping between the separation rate and the ratio of the job-finding rate when on the job (λ_1) and the exogenous displacement rate δ, $k_1 = \lambda_1/\delta$:

$$EE + \delta = \delta + \lambda_1 \int (1 - F(w))g(w)dw = \delta \ln(1 + k_1)\frac{(1 + k_1)}{k_1}.$$

We solve numerically for k_1 given the $(EE + \delta)/\delta$ ratio. Using $\hat{\lambda_1} = \hat{k_1}\hat{\delta}$ we can then pin down the job finding rate of the employed.

[13] The aggregate discount rate, β, is exogenously set to align with an annual capital return of 10 percent.

Firm Productivity Distribution: To set $\underline{p}$, we are guided by the model. We assume that the lowest productivity firm makes zero profits, such that $\underline{p} = \theta^R$. We then use the simulated method of moments with random samples to pin down α and b_0, which jointly determine the firm distribution given that $\theta^R = \theta^R(b_0)$. To do so we use the life-cycle wage profile to target the average wage changes at experience level 5-10 and 10-15 compared to age group 0-5 and minimize the squared percentage distance between these moments and the simulated data (at quarterly frequency). We focus on the first three age categories (i.e., 0-5, 5-10, and 10-15) since they correspond to the steepest part of the life-cycle wage profile and, hence, have the highest potential impact on workers' earnings over a lifetime.

On-the-job Learning:- We leverage the fact that job stayer wage growth is driven solely by learning on the job to estimate of the learning parameter, μ. Formally, we pin down this human capital accumulation parameter as:

$$\Delta w = \mu.$$

Table 4 summarizes the empirical moments used for estimation for the three study countries. In particular, we use the unemployment rate, the UE and EE rates, the job stayer wage growth rate and the average wage growth between age groups 0-5, 5-10 and 10-15. Note that we do not need to pin down the initial worker skill distribution as these do not affect the changes in average wages between years in our model. Our model hence relies on the estimation of relative moments alone.[14]

6 Results

We next present model-based evidence that illustrates the fundamental link between the distribution of firm productivity and life-cycle wage growth.

[14] Our estimation aims at contrasting the systematic change in average wages with age and the contributions of such systematic changes due to human capital evolution versus changes due to the job ladder. This is fundamentally different from estimating the variance contributions of worker and firm components in residualized wages, where systematic changes due to age are typically removed. An analysis of the variance contributions (as in Ozkan et al. (2023), Huggett et al. (2011), Guvenen et al. (2021) for the US) is beyond the scope of this paper and would require data moments that cannot be reliably obtained within any of our labor force surveys or in reliable quality across our sample data sets.

Table 4: Estimation Targets

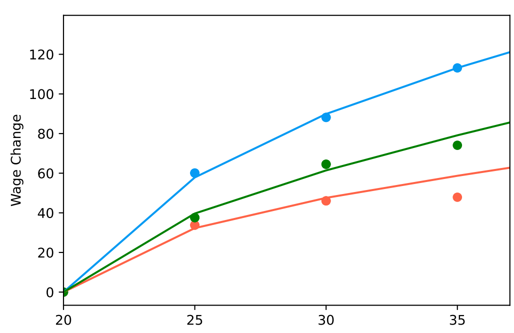
	Rates			Wage Growth		Human Capital
	Unemp. (u)	UE	EE	$w_{5/10} - w_{0/5}$	$w_{10/15} - w_{0/5}$	Growth (μ)
	(1)	(2)	(3)	(4)	(5)	(6)
United States	0.068	0.472	0.056	60.08	88.15	0.0095
Brazil	0.096	0.437	0.082	37.60	64.52	0.0088
Colombia	0.105	0.370	0.089	33.79	46.04	0.0056

Notes: This table presents the estimation targets used for estimation. u denotes the unemployment rate, UE denotes the unemployment to employment transition rate, EE denotes the job-to-job transition rate and $w_{10/15} - w_{0/5}$ and $w_{5/10} - w_{0/5}$ denote the average change in wages in age group 30-35 and 25-30 as compared to the reference age group 20-25, respectively. Human capital growth corresponds to the average change in log-wages for stayers under 32 years of age. Data for the United States comes from CPS, for Brazil from the PNADC, and for Colombia from household surveys All the rates are expressed in quarterly terms.

6.1 Model Estimates

We show the model predicted life-cycle wage profiles in Figure 4 together with the data from each of the countries of interest. The model aligns closely with the targeted moments of the life-cycle profiles, accurately capturing the differences in steepness across the three countries. Note that observations at age 25 and 30 are targeted in the estimation procedure, but not the observation at age 35.

Figure 4: Model Fit



Notes: The figure shows the average wage relative to the wage at age group 20-25 for empirical (dots) and simulated data (line) for the three countries. Data for the United States comes from CPS, data for Brazil from PNADC, and data for Colombia from household surveys and social security records. United States is shown in blue, Brazil is shown in green and Colombia in orange. Observations at age 25 and 30 are targeted in the estimation, but not the observation at age 35.

We present the estimation results in Table 5 for the parameters that govern the firm-type productivity distribution $(\alpha, \underline{p})$, labor market frictions $(\lambda_0, \lambda_1, \delta)$, and learning on the job (μ). Three key results emerge from these estimations. First, compared to the United States, both Brazil and Colombia exhibit a larger shape parameter (α) and a lower scale parameter $(\underline{p})$. A larger shape parameter suggests a smaller tail in the Pareto distribution of firm productivity, while a smaller scale parameter indicates a greater prevalence of low-quality jobs. These findings align with prior research, which shows that less developed countries tend to have fewer "superstar" firms and a shortage of well-paying jobs (Hsieh and Klenow, 2014; Eslava et al., 2022, 2023).

Table 5: Estimation Results

	Firm-type Distribution		Labor Market Frictions			Learning on the Job (μ)
	Shape (α)	Scale ($\underline{p}$)	δ	λ_1	λ_0	
United States	2.48	1.287	0.034	0.34	0.47	0.0095
Brazil	2.71	0.801	0.047	0.55	0.44	0.0088
Colombia	2.98	0.758	0.043	0.73	0.37	0.0056

Notes: This table presents the estimation results. α denotes the shape parameter of the Pareto distribution $\Gamma(p)$, b_0 determines the scale parameter of the Pareto distribution $\Gamma(p)$, δ denotes the exogenous separation rate, λ_1 and λ_0 denote the job finding rate when employed and unemployed, respectively, and μ denotes the rate of learning on the job.

Second, learning on the job (μ) is slightly higher in the United States than in Brazil and Colombia. This is consistent with the expectation that on-the-job training plays a more significant role in driving income profile differences in developed economies (Ma et al., 2024).

Third, regarding labor market frictions, separation rates (δ) are higher in both Colombia and Brazil than in the United States, suggesting that upward mobility through the job ladder is more challenging in these countries. This could slow wage growth from career advancement. More importantly, job-finding rates for the unemployed (λ_0) are lower in Brazil and Colombia compared to the United States, but job-finding rates for the employed (λ_1) are higher in Brazil and Colombia than in the United States.

These results for job-finding rates in the United States are consistent with previous studies showing that in developed countries, job-finding rates are lower for the employed than for the unemployed $(\lambda_1 < \lambda_0)$ (Bontemps et al., 2000; Bagger et al., 2014). However, the opposite relationship holding in Brazil and Colombia, where job-finding rates are higher among the employed $(\lambda_1 > \lambda_0)$ is novel to our analysis. This suggests a distinct pattern in less developed economies. To validate this finding, we use aggregated data from the cross-country labor force survey in Donovan et al. (2023), and plot the relationship between the difference in job-finding rates during employment and unemployment ($\lambda_1 - \lambda_0$) and countries' income in Appendix Figure A.3.[15] The data show that more developed countries tend to have a negative difference between λ_1 and λ_0 on average, while less developed countries exhibit a positive difference.

This finding is particularly significant because of its impact on the reservation wage component, θ^R, as detailed in Equation 2. In countries where it is easier to find a job while unemployed than while employed (i.e., more developed countries), workers tend to be more selective when evaluating job offers, resulting in a higher θ^R than the unemployment value. In contrast, in countries where it is easier to find a job while employed than while unemployed (i.e., less developed countries), workers are more inclined to accept lower-value jobs to advance in the job ladder. Consequently, the lower end of the job ladder in these countries tends to be longer, with a greater number of low-quality jobs available.

These observations contribute to a more nuanced understanding of the cross-country findings in Donovan et al. (2023), who identify a negative correlation between job-finding and job-exit rates and economic development. We confirm this relationship for the correlation between δ and λ_0, but we observe an opposite pattern for δ and λ_1. These results highlight a triple disadvantage for less developed countries in our sample: not only do they experience lower job-finding rates for the unemployed and higher job displacement rates for the employed, but the higher job-finding rates for the employed also suggest that entry-level jobs, on average, tend to be of lower quality.

6.2 Counterfactual Analysis

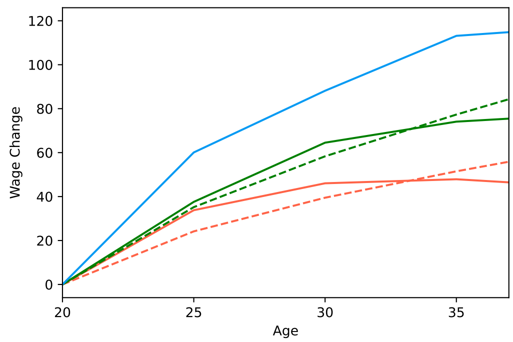
We use the model estimates to consider counterfactual scenarios for the life-cycle wage profile across countries. Specifically, we study separately the impact of: i) the firm productivity distribution; ii) labor market frictions; and iii) learning on the job. We vary one set of parameters at a time, while holding the remainder constant. Results are shown in Figure 5. Each plot varies one set of parameters for Brazil (in green) and Colombia (in orange), equating it to the United States' Level. Counterfactual estimates are then shown as dashed lines in the respective color. The benchmark estimate corresponds to the

[15] The data in Donovan et al. (2023) include unemployment-to-employment and job-to-job rates, along with GDP per capita. We supplement this data using unemployment rates from the International Labor Organization for the same periods.

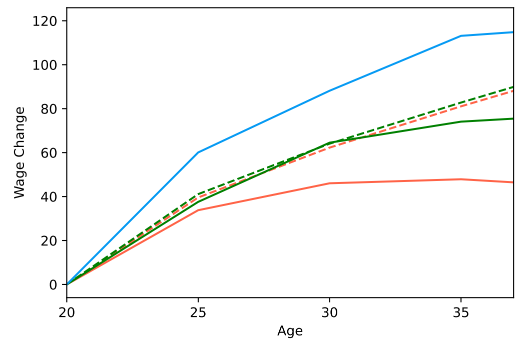
United States and is shown as solid blue line.

Figure 5: Counterfactual Exercise

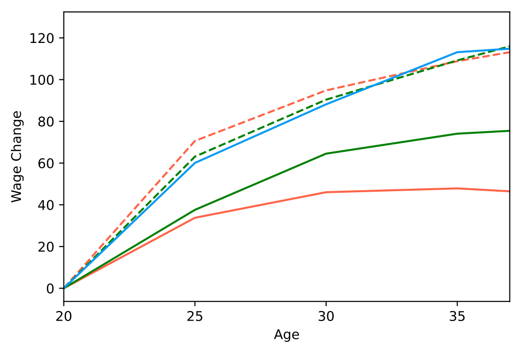
(a) Labor Market Frictions



(b) Learning on the Job



(c) Firm-Type Productivity Distribution



Notes: These figures show the average wage relative to the wage at age group 20-25 for both empirical and counterfactually simulated data. Data for the USA comes from CPS, data for Brazil from PNADC, and data for Colombia from household surveys and social security records. Each plot varies one aspect of the parameter space for Brazil (green) and Colombia (orange) compared to US estimates (blue). Counterfactual estimates are shown as dashed lines.

Figure 5a shows the life-cycle profile when adjusting the labor market friction parameters (λ_0, λ_1 and δ) to the United States level. A priori, these adjustments do not necessarily indicate improved access to the job ladder, as job-finding rates while employed are higher in Brazil and Colombia, despite the exogenous displacement rate being lower in the United States. Effectively, in the counterfactual analysis, these adjustments do not significantly alter the wage growth for neither Brazil nor Colombia.

Figure 5b considers the role of learning on the job. Increasing the human capital growth rate for Colombia to the United States level raises life-cycle wage growth to about the factual Brazilian level, while for Brazil no significant change pertains.[16]

Finally, Figure 5c examines an adjustment of both parameters of the firm-type distribution.[17] We observe a significant increase in the wage growth patterns for both Brazil and Colombia. The magnitude of the increase brings the wage growth for both countries closer to the levels seen in the United States, indicating a significant relevance of the firm-type distribution on wage progression.

These results are quantitatively summarized in Table 6, where we contrast the counterfactual evolution of average wages in Colombia and Brazil with that of the United States. Wage growth in both countries is expressed as the share of wage growth in the United States at age 30-35. We include benchmark measures in the first row indicating that wage growth in Colombia and Brazil at age 30-35 is equivalent to 52 and 73 percent, respectively, of the wage growth shown in the United States.

Table 6: Counterfactual Simulation

	Parameters changed to U.S. Level	Relative Wage Growth	
		Colombia	Brazil
	(1)	(2)	(3)
Share with Respect to U.S.		0.52	0.73
Counterfactual :			
- *Firm-Type Distribution*			
Shape	α	0.72	0.77
Scale	b_0	0.76	0.89
Shape and Scale	α, b_0	1.07	1.02
- *Learning on the Job*	μ	0.70	0.72
- *Labor Market Frictions*	$\lambda_1, \lambda_0, \delta$	0.44	0.66

Notes: This table presents the empirical value (first row) and three counterfactual estimation results (second to last row), expressed as the share of the wage growth in age bin 30-35 as compared to the value for the US economy. Each counterfactual changes the parameters in column (1) to match the parameter level of the United States. Columns (2) and (3) show the relative wage growth of the counterfactual Colombia (column (2)) and Brazil (column (3)) with respect to the United States benchmark.

A firm-type distribution with more productive firms seems to be the most determinant

[16] Note that due its effect on θ^R, a change in the human capital growth rate, μ, does not only change learning on the job directly but also impacts the wage growth pattern due to its effect on the lower end of the job ladder. For Brazil, the second effect is larger than the first and therefore leaves wage progression almost unchanged.

[17] Appendix Figures A.4b and A.4a illustrate changes in either the tail parameter α or the shape determinant b_0, separately.

component on raising the wage profiles of Brazil and Colombia. Our results suggest that matching the firm distribution of the United States will increase wage growth from 73 to 102 percent in Brazil and from 52 to 107 percent in Colombia, entirely closing the gap in the steepness of the wage growth trajectory over the life cycle across the three countries. Note that both the shape and the scale parameter are significant in achieving this, as both increase the relative wage growth closer to the United States level. Labor market frictions unambiguously decrease wage growth by reducing the access to more productive firms. However, matching the level of on-the-job growth will have sizable effects for Colombia, increasing it from 52 to 70 percent of the United States value, and no effect for Brazil.

6.3 Discussion

Our findings highlight the central role of the firm-type distribution in shaping differences in wage progression across countries. To set these results in perspective, we discuss the sensitivity of our conclusions to the model's structural assumptions. Two such assumptions are particularly relevant. *First*, we assume that wages are posted rather than bargained, as for instance in the sequential auction framework of Postel-Vinay and Robin (2002). *Second*, we assume that human capital accumulation is homogeneous across workers and independent of the firm rank. Even with departures from these two assumptions, our estimates represent a likely lower bound on the importance of the firm-type distribution within the class of labor search models that allow for separate estimation of labor market parameters and firm-type heterogeneity.

A key strength of our framework is the transparency it provides in linking estimation moments to parameter estimates. This is achieved by separately estimating the labor market parameters, the learning parameter and the parameters governing the firm productivity distribution. Specifically, the estimation of labor market parameters solely relies on worker mobility rates and the unemployment rate, the estimation of the learning parameter relies solely on wage growth of job stayers and the estimation of the firm type distribution achieves the best fit for wage growth conditional on labor market frictions and stayer wage growth.

This separation in the estimation procedure has two implications. First, any conclusion we draw regarding the importance of labor market frictions are independent of other parts of the model and notably the wage setting protocol. As a result, it becomes evident that any model preserving the mapping between worker mobility rates and the labor market parameters independently from the firm type distribution, will generate similar estimates of labor market frictions, when holding the search process constant. Second, our estimates of the firm type distribution's contribution to wage growth are conditional on observed wage growth among job stayers. Because these parameters are identified residually—after accounting for the learning parameter—their estimated importance declines as the learning rate rises. Hence, if stayers' wage growth is also influenced by the firm distribution (e.g., through outside offers), this would amplify rather than constrain the role of firm heterogeneity for wage growth.

This feature mitigates concerns related to the *first* assumption by preserving the robustness of labor market parameters under different wage bargaining structures. For instance, Moscarini and Postel-Vinay (2023) adopt a similar estimation strategy of separating out the estimation of labor market parameters but within a sequential auction framework. Hence, the estimated differences in labor market frictions across countries would remain unaffected irrespective of the model's wage bargaining assumptions, assuming the same search process. Moreover, among standard models that allow for this separation, the importance of the firm-type distribution relative to human capital accumulation is unlikely to diminish when compared to our result—if anything, it may be amplified. For instance, consider on-the-job wage renegotiation as in Postel-Vinay and Robin (2002). If wage renegotiation were permitted, some of the wage growth we currently attribute to learning would instead reflect rent increases, further strengthening the role of firm heterogeneity across the job ladder for wage growth. Hence, our results provide a lower bound on the role of firm productivity distributions in explaining cross-country differences in life-cycle wage profiles compared to such frameworks.

Moreover, we view wage posting as a more empirically relevant modeling assumption in our context. Naturally, it is *a priori* unclear whether wages are primarily set through bargaining or wage posting. Recent empirical evidence from surveys and structural estimations indicates that over 60 percent of firms in Austria, Germany, and the US use wage posting (Holzheu and Robin, 2024; Flinn and Mullins, 2019). In addition, wage posting has been shown to be particularly prevalent among low-skill jobs (Brenčič, 2012; Brenzel et al., 2014), which supports its relevance in our setting. While wage posting is not universal, its prevalence in our sample's context—particularly among lower-skilled positions—makes it a plausible and empirically grounded assumption. Given this evidence, we consider that our findings are the most likely estimate in our empirical setting.

Regarding the *second* assumption—that human capital accumulation is independent of firm productivity—we note that allowing for heterogeneity in learning rates across firms would likely reinforce our main result on the role of firms in driving wage growth. We maintain this assumption for tractability and alignment with standard practice in the labor search literature (e.g., Bagger et al. (2014)). Nonetheless, recent evidence points to variation in on-the-job learning across firms (Arellano-Bover, 2021; Gregory, 2020). In particular, Arellano-Bover (2024) finds a positive relationship between firm size—a common proxy for productivity—and learning rates. Omitting this mechanism could bias our estimates downward, as it excludes a potential amplification channel through which firm productivity differences translate into wage dispersion.

These considerations suggest that our estimates provide a likely lower bound on the contribution of the firm-type distribution to cross-country differences in wage growth, within the class of models that allow to separately identify labor market parameters and firm heterogeneity.

7 Conclusion

In this paper, we examine how the local distribution of firm productivity shapes life-cycle wage profiles across the United States, Brazil, and Colombia. We introduce a labor market random search model that allows to disentangle the role of the firm-type distribution, labor market frictions and on-the-job learning in influencing workers' life-cycle wage growth. Our empirical analysis suggests that the share of workers employed in larger firms— a reasonable proxy for the firm productivity distribution, as indicated by the model and prior empirical evidence— significantly affects differences in life-cycle wage profiles both across and within these countries. We then estimate the model and find that variations in the distribution of firm types have the greatest impact on the steepness of life-cycle wage profiles. This finding complements previous research on cross-country differences in life-cycle wage growth trajectories.

Our results should be interpreted as lower bounds for the importance of labor market opportunities in shaping life-cycle wage profiles. Our model interprets wage growth rates of stayers as indicative of on-the-job learning. However, such wage growth could also reflect wage bargaining through outside offers by other firms (Postel-Vinay and Robin, 2002). In such a scenario, on-the-job learning would account for a smaller share of life-cycle wage growth. Moreover, while our model assumes a constant rate of learning on-the-job, it is conceivable that more productive firms offer higher learning rates (Gregory, 2020), which would increase the importance of a long right tail of the firm type distribution.

Our findings indicate that steeper life-cycle income profiles—often associated with higher levels of economic development—may depend on the presence of a thicker right tail in the firm productivity distribution. This insight complements existing research comparing firm productivity distributions across countries at different stages of development (Hsieh and Klenow, 2009, 2014). We also highlight the crucial role of the job ladder in improving labor market outcomes for workers across countries. Trade policies that allow foreign firms to enter local labor markets could expand the right tail of the job ladder, thus providing more attractive labor market opportunities for workers. Therefore, trade openness can contribute to steeper life-cycle wage profiles for workers and is a topic still to be explored.

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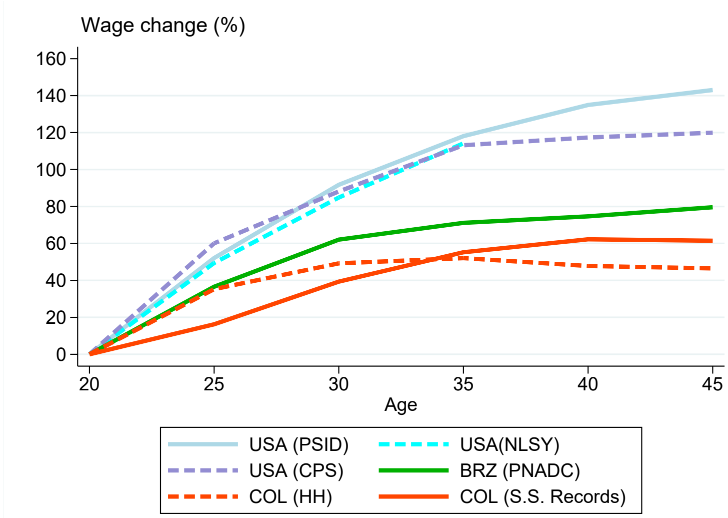
Appendix

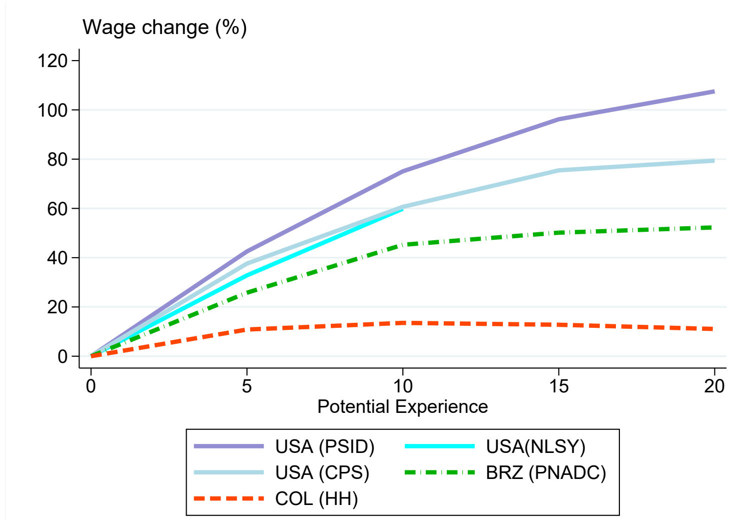
A Appendix Figures and Tables

Appendix Figure A.1: Age Profile of Wages Across Multiple Data sets



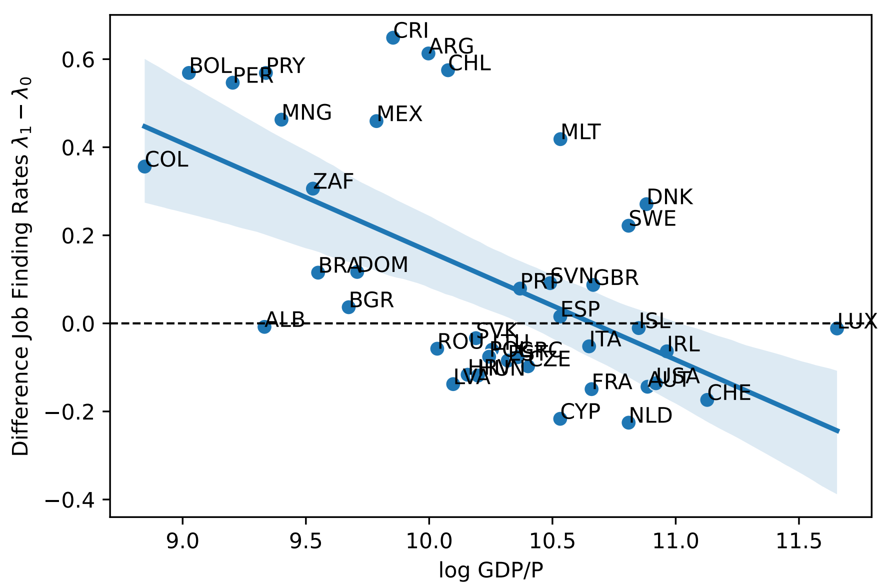
Notes: Wage change corresponds to the growth rate of average monthly wages for workers in each 5-year age bins relative to the average wages of workers between the ages of 20 and 24.

Appendix Figure A.2: Experience Profile of Wages Across Multiple Data Sets



Notes: Wage change corresponds to the growth rate of average monthly wages for workers in each 5-year experience bins relative to the average wages of workers with 0 to 4 years of experience. Potential experience as the lesser of two measurements of duration: the time since reaching the age of 18 or since graduating from the highest level of education.

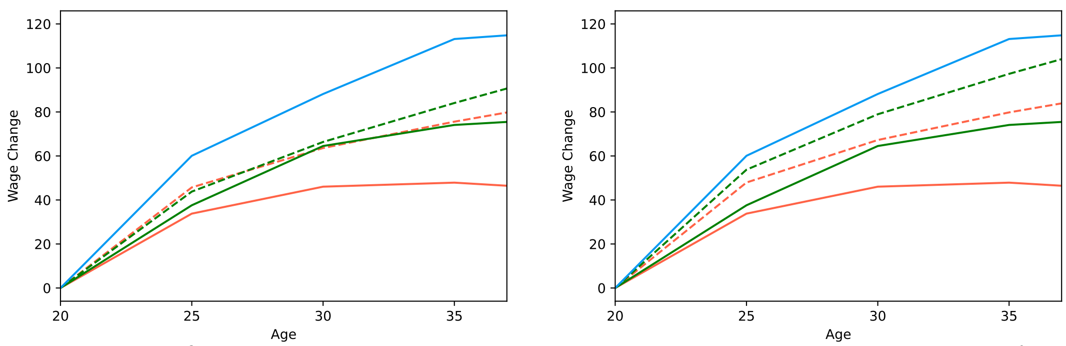
Appendix Figure A.3: Difference in Job Finding Rates and Economic Development



Notes: The figure represent the relationship between the job finding rate for the employed (λ_1) and the job finding rate for the unemployed (λ_0) across countries. Data derived from Donovan et al. (2023) on UE and EE rates as well as GDP per capita together with unemployment rates from the International Labor Organisation. We then use our identification procedure to back out λ_1 and λ_0.

Appendix Figure A.4: Counterfactual Exercise

(a) Firm Distribution Shape Parameter α (b) Firm Distribution Scale Parameter b_0



Notes: These figures show the average wage relative to the wage at age group 20-25 for both empirical and counterfactually simulated data. Data for the USA comes from CPS, data for Brazil from PNADC, and data for Colombia from om household surveys and social security records. Each plot varies one aspect of the parameter space for Brazil (green) and Colombia (orange) compared to US estimates (blue). Counterfactual estimates are shown as dashed lines.

B Mathematical Details

B.1 Firm size

In steady state, the number of workers leaving a firm $S(w_0(p))$ and new number of new recruits $R(w_0(p))$ at the firm has to balance such that $S(w_0(p)) = R(w_0(p))$. Note that we can write this equation as $s(w_0(p))l(w_0(p)) = R(w_0(p))$, where $s(w_0(p))$ is the worker exit rate from the firm. Hence, firm size $l(w_0(p))$ is obtained as:

$$l(w_0(p)) = \frac{R(w_0(p))}{s(w_0(p))}.$$

Recall that exit rate $s(w_0(p))$ is obtained as the sum of displacement and E2E mobility, $s(w_0(p)) = \delta + \lambda_1 \overline{F}\left(w_0(p)\right))$. We now derive the number of new recruits $R(w_0(p))$.

A firm that pays $w_0(p)$ recruits workers from two sources: 1. from the unemployed $(\lambda_0 u)$ and 2. from employed workers earning less than $w_0(p)$ $((1-u)\lambda G(w_0(p)))$. The total flow of recruits to the firm, $R(w_0(p))$ is then

$$\begin{aligned}
R(w_0(p)) &= (u\lambda_0 + (1-u)\lambda_1 G(w_0(p))) \\
&= \left(\frac{\delta\lambda_0}{\delta+\lambda_0} + \frac{\lambda_1\lambda_0}{\delta+\lambda_0}\frac{\delta F(w_0(p))}{\delta+\lambda_1\overline{F}\,(w_0(p)))}\right) \\
&= \left(\frac{\lambda_0\delta(\delta+\lambda_1)}{(\delta+\lambda_1\overline{F}\,(w_0(p))))(\delta+\lambda_0)}\right) \\
&= \left(\frac{\lambda_0\delta}{(\delta+\lambda_1\overline{F}\,(w_0(p))))}\right)
\end{aligned}$$

In this derivation we use two equations. First, we use the equilibrium relationship for unemployment $u = \frac{\delta}{\delta+\lambda_0}$. Second, we use the steady-state equation balancing the density of workers at wage component $w_0(p)$ through inflows from unemployment $u\lambda_0 F(w_0(p))$ and outflows to either non-employment or higher paying firms $(1-u)(\delta+\lambda_1\overline{F}\,(w_0(p))))$. We can hence write the relationship between the distribution of wages across workers $G(w_0(p))$ and the distribution across firms $F(w_0(p))$ as:

$$G(w_0(p)) = \frac{\delta F(w_0(p))}{\delta+\lambda_1\overline{F}\,(w_0(p)))}$$

Finally, bringing together $s(w_0(p))$ and $R(w_0(p))$, we obtain for the firm size $l(w_0(p))$

$$\begin{aligned}
l(w_0(p)) &= \frac{R(w_0(p))}{s(w_0(p))} \\
&= \frac{\lambda_0\delta}{\left(\delta+\lambda_1\overline{F}\,(w_0(p)))\right)^2}
\end{aligned}$$

Using the replacement $k_0 = \lambda_0/\delta$ and $k_1 = \lambda_1/\delta$, we can write this as

$$l(w_0(p)) = \frac{k_0}{\left(1+k_1\overline{F}\,(w_0(p)))\right)^2}.$$

B.2 Derivation Reservation Wage Component

Using integration by parts and incorporating the reservation strategy, we can write the two value functions as:

$$U(h) = b(h) + \beta U(h) + \beta\lambda_0 \int_{\theta^R}^{\overline{w}} W_x \overline{F}(x)dx \tag{B.1}$$

$$W(w_0,h) = w + \beta\lambda_1 \int_{\theta^R}^{\overline{w}} W_x \overline{F}(x)dx + \beta\delta U(h) + \beta(1-\delta)W(w_0,h') \tag{B.2}$$

Differentiating the worker value function with respect to w_0 yields:

$$W_w(w_0, h) \;=\; 1 + \beta(1 - \delta - \lambda_1 \overline{F}(w)) W_w(w_0, h').$$

Solving for $W_w(w_0, h)$ we find that[18]:

$$W_w(w_0, h) \;=\; \frac{1}{1 - \beta(1 - \delta - \lambda_1 \overline{F}(w_0))}. \tag{B.3}$$

This implies that the option value of search does not depend on a worker's human capital. We pin down the reservation wage component, θ^R, by combining the wage equation in (1) with the value functions in (B.1) and (B.2) and the equation for unemployment benefits. This combination implies that:

$$b_0 - \theta^R \;=\; \beta(1-\delta)\left(W(\theta^R, h') - U(h')\right) + \beta(1-\delta)\left(U(h') - U(h)\right) \tag{B.4}$$

$$+ \;\; \beta(\lambda_0 - \lambda_1) \int_{\theta^R}^{\overline{w}} W_x \overline{F}(x) dx \tag{B.5}$$

We proceed to guess and verify an equilibrium with equal reservation wage component for all workers, such that $\partial\theta^R/\partial h = 0$. In this case, using the equation for $U(h)$ together with the result in equation B.3 that $\frac{\partial W_{w_0}}{\partial h} = 0$, we find that

$$U(h') - U(h) = \frac{h' - h}{1-\beta} = \frac{\mu}{1-\beta}.$$

Together with the equation for $W(\theta^R, h) - U(h)$, this implies that the reservation wage component is[19]:

$$\theta^R \;=\; b_0 - \mu\frac{\beta(1-\delta)}{1-\beta} + \beta(\lambda_0 - \lambda_1) \int_{\theta^R}^{\overline{w}} W_x \overline{F}(x) dx$$

[18] Note that this result is identical as in Bagger et al. (2014).

[19] To see that this is indeed an equilibrium, differentiate equation 2 with respect to h to obtain

$$\frac{\partial\theta^R}{\partial h}(-1 - \beta(1-\delta)W_{\theta^R}) = \beta(1-\delta)\left(W_h - U_h\right)$$

For $\frac{\partial\theta^R}{\partial h} = 0$ we require that $W_h = U_h$. In this case, $U_h = \frac{1}{1-\beta}$ and $W_h = 1 + \beta\delta U_h + \beta(1-\delta)W_h = \frac{1}{1-\beta}$

B.3 Wage equation

The envelope theorem implies the equilibrium condition

$$\pi'(p) = \frac{1}{\left[\delta + \lambda_1 \overline{F}\left(w_0(p)\right)\right]^2}$$

or

$$\pi(p) - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda_1 \overline{\Gamma}\left(w_0^{-1}\right)\right]^2} dx$$

where $w_0^{-1} = q$ denotes the inverse function to $w_0(p)$. We also denote the first-order condition

$$2(p - w_0(p)) \frac{\lambda f(w_0)}{\delta + \lambda_1 \overline{F}\left(w_0\right)} = 1.$$

It follows from the equilibrium condition that

$$\frac{p - w_0(p)}{\left[\delta + \lambda_1 \overline{\Gamma}\left(w_0(p)\right)\right]^2} - \pi(\underline{p}) = \int_{\underline{p}}^{p} \frac{1}{\left[\delta + \lambda_1 \overline{\Gamma}\left(x\right)\right]^2} dx. \tag{B.6}$$

B.4 Comparative Statics

We study the change in the slope of the wage-productivity schedule with respect to some parameter ρ, that is $\frac{\partial^2 w}{\partial p \partial \rho}$. To do this, we leverage the first order condition with equilibrium condition $F(w_0(p)) = \Gamma(p)$

$$2(p - w_0(p)) \frac{\lambda_1 \gamma(p)}{\delta + \lambda_1 \overline{\Gamma}\left(p\right)} = 1$$

using the parametric assumption $\Gamma(p) = 1 - \left(\frac{\underline{p}}{p}\right)^{\alpha}$. By the implicit function theorem,

$$\left(1 - \frac{\partial w}{\partial p}\right) - (\alpha + 1) \frac{\delta + \lambda_1 \overline{\Gamma}\left(p\right)}{\lambda_1 \gamma(p)} - 1/2 = 0$$

Hence, we know that

$$
\begin{aligned}
\frac{\partial^2 w}{\partial p \partial \lambda_1} &= (\alpha+1)\frac{\delta p^{\alpha+1}\underline{p}^{-\alpha}}{\lambda_1 \alpha} > 0 \\
\frac{\partial^2 w}{\partial p \partial \delta} &= -(\alpha+1)\frac{p^{\alpha+1}\underline{p}^{-\alpha}}{\lambda_1 \alpha} < 0 \\
\frac{\partial^2 w}{\partial p \partial \underline{p}} &= (\alpha+1)\frac{\delta p^{\alpha+1}\underline{p}^{-\alpha}}{\lambda_1} > 0 \\
\frac{\partial^2 w}{\partial p \partial \alpha} &= (\alpha+1)\left(-\left(\frac{p^{\alpha+1}\underline{p}^{-\alpha}(\delta+\lambda_1 \frac{\underline{p}}{p}^{\alpha})}{\lambda_1 \alpha^2}\right) - \left(\frac{\log(p) p^{\alpha+1}\underline{p}^{-\alpha}(\delta+\lambda_1 \frac{\underline{p}}{p}^{\alpha})}{\lambda_1 \alpha}\right)\right) \\
&- (\alpha+1)\left(\left(\frac{\log(\underline{p}) p^{\alpha+1}\underline{p}^{-\alpha}(\delta+\lambda_1 \frac{\underline{p}}{p}^{\alpha})}{\lambda_1 \alpha}\right) - \left(\frac{p^{\alpha+1}\underline{p}^{-\alpha}\frac{\underline{p}}{p}^{\alpha}\log(\underline{p}/p)}{\alpha}\right)\right) \\
&= -(\alpha+1)\cdot \frac{p^{\alpha+1}\underline{p}^{-\alpha}(\delta+\lambda_1(\underline{p}/p)^{\alpha})}{\lambda_1 \alpha^2}(1+2\alpha \log p) >< 0
\end{aligned}
$$

Specifically, $\frac{\partial^2 w}{\partial p \partial \alpha} < 0$ for larger values of productivity $p > e^{1/(2\alpha)}$ and positive for smaller values $p < e^{1/(2\alpha)}$.

B.5 Distribution of Workers

Using the functional form assumption

$$
\Gamma(p) = F(p) = 1 - \left(\frac{\underline{p}}{p}\right)^{\alpha}
$$

and the equilibrium distribution of workers at firm quality p

$$
G(p) = \frac{\delta F(p)}{\delta + \lambda_1(1-F(p))}
$$

we are interested in

$$
\frac{\partial \overline{G}(p)}{\partial \alpha}, \quad \frac{\partial \overline{G}(p)}{\partial \underline{p}}
$$

First, using

$$
\overline{G}(p) = 1 - G(p) = \frac{\delta + \lambda_1 - (\lambda_1+\delta)\left[1-\left(\frac{\underline{p}}{p}\right)^{\alpha}\right]}{\delta + \lambda_1\left(\frac{\underline{p}}{p}\right)^{\alpha}}
$$

and

$$\frac{\partial F(p)}{\partial \alpha} = -\left(\frac{\underline{p}}{p}\right)^{\alpha} \ln\left(\frac{\underline{p}}{p}\right)$$

We obtain

$$\begin{aligned}
\frac{\partial \overline{G}(p)}{\partial \alpha} &= \frac{-(\lambda_1+\delta)\frac{\partial F}{\partial \alpha}(\delta+\lambda_1\overline{F}(p)) - (\delta+\lambda_1-(\lambda_1+\delta)F(p))(-\lambda_1\frac{\partial F(p)}{\partial \alpha})}{(\delta+\lambda_1\overline{F}(p))^2} \\
&= \frac{\left(\frac{\underline{p}}{p}\right)^{\alpha} \ln\left(\frac{\underline{p}}{p}\right)\left[(\lambda_1+\delta)(\delta+\lambda_1\overline{F}(p)) + \lambda_1(\delta+\lambda_1-(\lambda_1+\delta)F(p))\right]}{(\delta+\lambda_1\overline{F}(p))^2} > 0
\end{aligned}$$

Next, using

$$\frac{\partial F(p)}{\partial \underline{p}} = \alpha\left(\frac{\underline{p}}{p}\right)^{\alpha-1}\frac{1}{p}$$

we obtain:

$$\frac{\partial \overline{G}(p)}{\partial \underline{p}} = \frac{\alpha\left(\frac{\underline{p}}{p}\right)^{\alpha-1}\frac{1}{p}\left[(\lambda_1+\delta)(\delta+\lambda_1\overline{F}(p)) + \lambda_1(\delta+\lambda_1-(\lambda_1+\delta)F(p))\right]}{(\delta+\lambda_1\overline{F}(p))^2} > 0$$

Hence, the density of workers at firms with productivity of at least p $\overline{G}(p)$ is increasing in the lower support of the distribution $\underline{p}$ and the tail parameter of the distribution α.

B.6 Exit Rate

A worker's likelihood of exiting a firm with wage component w_0, $qr(w_0)$, is composed of the exogenous likelihood of separating, δ and the likelihood of job mobility. The latter depends on the wage offer w_0 and the wage offer distribution $F(w_0)$, such that any draw from the job offer distribution exceeding w_0 will be accepted (occurring at rate $(1-F(w_0))$), conditional on receiving a job offer at rate λ. The compound likelihood of job mobility is hence $\lambda(1-F(w_0))$. Together, we write the likelihood of exiting as

$$qr(w_0) = (\delta + \lambda_1(1 - F(w_0)))$$

The expectation of the job exit rate is then

$$
\begin{aligned}
E[qr] &= \delta + \lambda_1 \int_{\underline{w}}^{\overline{w}} (1 - F(w))g(w)dw \\
&= \delta + \lambda_1 \int_{\underline{p}}^{\infty} (1 - \Gamma(p(w)))g(w(p))w_p dp \\
&= \delta + \lambda_1 \int_{\underline{p}}^{\infty} \frac{\bar{\Gamma}(p)\gamma(p)(1+k_1)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} \frac{1}{\frac{dw}{dp}} w_p dp \\
&= \delta + \lambda_1 \int_{\underline{p}}^{\infty} \frac{\bar{\Gamma}(p)(1+k_1)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} d\Gamma(p) \\
&= \delta - \lambda_1 \int_{\underline{p}}^{\infty} \frac{\bar{\Gamma}(p)(1+k_1)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} d\bar{\Gamma}(p)
\end{aligned}
$$

where we use the equilibrium relationship $F(w) = \Gamma(p(w))$ and the fact that the wage offer distribution $F(w)$ is related to the distribution of workers across contracts $G(w)$ (cf. Bontemps et al. (2000), p. 310).[20] We finally use the substitution $\bar{\Gamma}(p) = s$ with bounds of integration [1,0].

$$
\begin{aligned}
E[qr] &= \delta - \lambda_1 \int_{\underline{p}}^{\infty} \frac{\bar{\Gamma}(p)(1+k_1)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} d\bar{\Gamma}(p) \\
&= \delta - \lambda_1(1+k_1) \int_{1}^{0} \frac{s}{(1+k_1 s)^2} ds = \delta + \lambda_1(1+k_1) \int_{0}^{1} \frac{s}{(1+k_1 s)^2} ds
\end{aligned}
$$

[20]We deduce from

$$
\begin{aligned}
1 + k_1 G(w) &= \frac{1+k_1}{1+k_1\bar{F}(w)} \\
\left(1+k_1\bar{F}(w)\right) k_1 G(w) &= (1+k_1) - \left(1+k_1\bar{F}(w)\right) = k_1 F(w)
\end{aligned}
$$

Given equilibrium constraint $F(w(p)) = \Gamma(p(w))$

$$
G(w(p)) = \frac{\Gamma(p(w))}{1+k_1\bar{\Gamma}(p(w))}
$$

Hence

$$
G'(w(p)) = \frac{\left(\gamma(p)(1+k_1\bar{\Gamma}(p)) + k_1\gamma(p)\Gamma(p)\right)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} \frac{1}{\frac{dw}{dp}} = \frac{\gamma(p)(1+k_1)}{\left(1+k_1\bar{\Gamma}(p)\right)^2} \frac{1}{\frac{dw}{dp}}
$$

We will now solve for the integral $I = \int_0^1 \frac{s}{(1+k_1 s)^2}\, ds$. First, let $u = 1 + k_1 s$, such that then: $du = k_1\, ds$ and $s = \frac{u-1}{k_1}$. Substitute these into the integral:

$$I = \int_0^1 \frac{\frac{u-1}{k_1}}{u^2} \cdot \frac{du}{k_1} = \frac{1}{k_1^2} \int_0^1 \frac{u-1}{u^2}\, du = \frac{1}{k_1^2} \int_0^1 \left(\frac{1}{u} - \frac{1}{u^2} \right) du.$$

We can now integrate each term and finally back-substitute, such that

$$I = \left(\frac{1}{k_1^2} \left(\ln |u| + \frac{1}{u} \right) \right) \Bigg|_0^1 = \left(\frac{1}{k_1^2} \left(\ln |1 + k_1 s| + \frac{1}{1 + k_1 s} \right) \right) \Bigg|_0^1 = \frac{1}{k_1^2} \left(\ln |1 + k_1| - \frac{k_1}{1 + k_1} \right)$$

We now bring back together the equation

$$\begin{aligned} E[qr] &= \delta + \lambda_1 (1 + k_1) \int_0^1 \frac{s}{(1 + k_1 s)^2} ds \\ &= \delta + \lambda_1 (1 + k_1) \frac{1}{k_1^2} \left(\ln(1 + k_1) - \frac{k_1}{1 + k_1} \right) = \delta \ln(1 + k_1) \frac{(1 + k_1)}{k_1} \end{aligned}$$

Note that this expression is independent of the specific nature of the firm-type distribution.