The Role of Workers in Knowledge Diffusion Across Firms*

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Abstract

We analyze the effect of labor mobility and innovation on productivity growth. We develop a multi-worker-firm framework with random search and on-the-job mobility to estimate the relative contribution of mobility and R&D to growth. With event-study analysis based on shift-share international trade shocks, we show that extent and direction of worker mobility affect firm productivity. Using Swedish microeconomic data, we find that over 44% of growth in average firm productivity can be attributed to worker mobility. Our results suggest that a slowdown of worker mobility, and more mobility from less productive firms, can depress growth in average firm productivity.

JEL codes: J24, J41, J63, J64
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1 Introduction

Understanding how firms increase their productivity, and how this translates into aggregate productivity, is central to macroeconomic analysis. The canonical answer emphasizes innovation through in-house research and development (R&D): firms invest in R&D, discover new ideas, and become more efficient in production. Yet in practice, in-house R&D is limited and highly concentrated in a small subset of firms (Klette and Kortum, 2004). In contrast, almost all firms hire new workers, and workers carry knowledge and ideas that can improve firm performance. In the United States, roughly one-fifth of all jobs are created or destroyed each year (Leonard, 1986; Davis and Haltiwanger, 1992), implying substantial reallocation of workers—and, with them, of ideas and experience—across firms. These patterns suggest that worker mobility and innovation interact in shaping productivity. At the aggregate level, they are likely to be complements: knowledge created through R&D spreads more widely when workers move, and mobility-induced diffusion amplifies the economy-wide returns to innovation. Consequently, labor mobility may be an important contributor to productivity growth.

In this paper, we quantify the role of workers in driving economic growth. We make three main contributions. First, we develop a novel multi-worker-firm environment with on-the-job search, building on Lucas and Moll (2014), Bilal et al. (2022) and Lamadon et al. (2024), that enables us to disentangle the contributions to economic growth arising from labor mobility and firm-generated innovation. This framework is distinctive in its allowance for the interplay of heterogeneous worker and firm productivity while remaining sufficiently parsimonious to ensure tractability. Our model conceptualizes firms as productive communities with shared histories, shaped by learning and innovation events as well as worker mobility. This approach allows us to interpret knowledge diffusion as the result of intra-firm learning driven by labor mobility and R&D, facilitating a closer alignment between model elements and empirical data as compared to the first contributions on productivity diffusion (Lucas and Moll, 2014). Our model stands out in allowing for job-to-job (EE) mobility from high to low quality firms in a multi-worker-firm environment, an arguably salient factor in worker mobility patterns that is notoriously difficult to capture. As a second contribution, we provide empirical evidence of the effect of extent and direction of worker mobility on firm productivity, leveraging highly granular register data for Sweden on firms, products and workers. We consider both vertical productivity changes, such as increases in output per worker, as well as horizontal productivity changes such as adoption of new products. Our data's granularity allows us to consider arguably exogenous shifters to tease out the effect of worker mobility. Finally, we estimate the aggregate implications of worker mobility, drawing on comprehensive data from the Swedish manufacturing sector, where innovation arguably has a more pronounced influence compared to other economic sectors. Our findings highlight the amplifying role of worker mobility in fostering economic growth. Quantitatively, we estimate that worker mobility accounts for over 44% of growth in average firm productivity in the manufacturing sector. In particular, job-to-job transitions account for the largest share of the rise in average firm productivity. We calibrate our model using mobility patterns (reminiscent of Hotz and Miller, 1993) and show in counterfactual exercises that the composition of worker flows plays a significant role for growth and is as large as the contribution of reducing mobility frictions. The contributions are quantitatively significant: reducing Swedish employment protection legislation to the level of Australia (in 2010) would increase average aggregate growth by about 1 percentage point. Taken together, these findings highlight the central importance of worker mobility for understanding aggregate growth.

In our theoretical framework, we conceptualize workers as vessels of knowledge. Knowledge raises output per worker, reflecting the presence of transferable, general human capital.[1] Firms generate new ideas through R&D and transmit these ideas to their workers, thereby assuming a dual role as both innovators and knowledge disseminators. Worker mobility, then, acts as an amplification mechanism of R&D efforts in the economy. Unlike most labor-search frameworks with multi-worker firms, where workers coexist in production without interacting (e.g., Moscarini and Postel-Vinay, 2013; Grübener and Rozsypal, 2024; Gulyas, 2024), our model allows the mobility of one worker to affect the marginal productivity of others within the same firm, adding substantial complexity. As a result, the extent and direction of labor mobility emerge as central features of our framework. To the best of our knowledge, ours is the first theoretical framework to integrate knowledge spillovers in multi-worker firms with on-the-job search. This is particularly important given that, in many advanced economies, job-to-job flows account for the majority of hires and separations—often exceeding flows from unemployment by a factor of 2 to 3. This empirical pattern underscores the fact that most workers who change jobs do so without experiencing a spell of unemployment, making it essential for our framework to capture this empirical pattern. A key mechanism in our model is that, within our multi-worker-firm environment with constant returns, workers are willing to move not only to more productive firms but also to less productive ones. This mechanism enables firms to acquire knowledge from other firms through on-the-job

[1] Worker movements among top tech firms and their anecdotal press coverage are real-world examples of worker mobility events spurring general knowledge transfer. For instance, Vox wrote in 2016 "New Apple hire is probably a sign that 'Tesla's graveyard' will eventually be a threat to Tesla" (Bhuiyan, 2016). Another news outline summarized a trajectory as "Ex-Tesla VP turned Apple Car engineer poached by electric plane startup in latest staff loss" (Hall, 2021).

search, bypassing the need to wait for highly knowledgeable workers to become unemployed. Our model rationalizes such moves and shows that they are equilibrium outcomes. As a result, our model resolves a key trade-off present in frameworks that lack job-to-job mobility but allow for knowledge transfers. In such frameworks, unemployment is the primary channel for reallocating knowledge across the economy, creating an inherent tension between knowledge diffusion and worker inactivity. In contrast, our model eliminates this trade-off by facilitating continuous knowledge flow through worker mobility on the job. This mechanism also captures an important empirical feature of worker mobility—specifically, downward worker mobility from more to less productive firms (cf. Sorkin, 2018; Holzheu and Robin, 2025).

We provide empirical evidence in support of our mechanism, using both suggestive aggregate evidence by comparing sectors of the economy and quasi-experimental evidence at the individual worker and firm level. Consistent with key predictions of our model, we find that sectors with more worker mobility, higher diversity of the worker composition and those who tend to hire more often from more productive firms see higher aggregate productivity growth. We then leverage quasi-experimental variation to argue for a causal interpretation to these data patterns. We exploit exogenous trade demand shocks to local peer firms to analyze the effect of shifts in the distribution of newly hired workers on local firm productivity. As negative trade shocks reallocate workers from one firm to another, we observe arguably exogenous variation in likely sending firms. We find that it matters which firm is hit by such shocks: if above median productivity firms are exposed to negative demand shocks, local competitors see increases in labor productivity on impact, whereas this is not true when a lower quality firm is hit. We use this quasi-experimental variation to pin down a key parameter in our model, the knowledge adoption intensity upon worker mobility. We find a model-consistent discontinuity around hiring from more productive as compared to less productive firms. This evidence adds to the previous literature on local effects of trade-based demand shocks, by showing that labor market spillovers can have heterogeneous effects depending on the origin firms.

Our findings have clear implications for productivity growth in light of the secular decline in worker mobility across firms and regions and persistent cross-country gaps. Holding other factors fixed, the model implies that lower labor mobility reduces aggregate productivity growth. This has direct policy relevance: institutions that damp mobility—such as employment protection—can depress aggregate productivity independently of their effects on firms' R&D incentives (cf. Aghion et al., 2023). We find that increasing knowledge adoption or raising contact rates for both high- and low-productivity firms raises growth, and that the immediate growth contribution from new firm creation is smaller than that from labor mobility, underscoring the importance of new-firm hiring for aggregate growth. Moreover, greater dispersion in workers' mobility valuations raises growth directly, apart from any innovation benefits associated with worker diversity. Our results are particularly relevant in downturns, when displacements increase the share of workers moving from lower-quality firms. By compressing growth-enhancing mobility flows, our model predicts that these reallocations further amplify productivity declines.

This paper contributes to three main strands of the literature. The first concerns the role of knowledge creation and diffusion in driving economic growth. In Schumpeterian theories of creative destruction, worker mobility reallocates labor from less to more productive firms as entry and exit occur (Aghion and Howitt, 1992; Lentz and Mortensen, 2008; Acemoglu et al., 2018). We build on this framework by documenting and quantifying a distinct mechanism: mobile workers transmit knowledge across firms. These flows play a first-order role in aggregate productivity growth by directly raising the productivity of receiving firms. Yet worker mobility is subject to frictions. Building on Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), prior work shows that barriers to reallocating resources toward productive firms depress aggregate productivity. Peters (2020) further shows that such misallocation reduces firms' incentives to innovate. Our mechanism highlights a complementary mechanism: frictions that hinder the mobility of knowledgeable workers constrain firms' abilities to grow in productivity, thereby slowing aggregate growth. Finally, by viewing workers as carriers of knowledge, our theory provides a specific interpretation for the spillovers central to endogenous growth models (Romer, 1990; Griliches, 1992; Jaffe et al., 1993; Jones, 1995; Lucas and Moll, 2014; Benhabib et al., 2021). We identify worker mobility as a key channel through which such spillovers occur, linking knowledge diffusion to salient features of the labor market—such as search efficiency and policies that restrict mobility. Frictions that impede reallocation thus distort not only the allocation of labor but also the generation of knowledge spillovers that underlie long-run growth.

The second strand of the literature has advanced on the theoretical study of either multi-worker firms or models of idea diffusion. Our paper is closely related to Bilal et al. (2021, 2022), Engbom (2023) and Lamadon et al. (2024), yet introduces worker heterogeneity and allows some simplifications compared to their setups. It shares with Jarosch et al. (2021) the focus on learning in firms, but does not consider wage formation, similar to Bilal et al. (2022). In addition, we use exogenous variations in the data to estimate key model parameters. Thematically, the paper is related to Lentz and Mortensen (2022), yet does not feature a product quality ladder. Our paper is the first to allow knowledge diffusion via job-to-job mobility. The paper also relates to Kortum (1997), Luttmer (2007), Lucas and

Moll (2014), Perla and Tonetti (2014), Koenig et al. (2016), Buera and Oberfield (2020), Hopenhayn et al. (2020), Benhabib et al. (2021), Bradley and Gottfries (2022) and Liu (2023) through its analysis of an economy with knowledge diffusion. While these papers mostly abstract from the mechanism behind idea diffusion, we explicitly bring to the model both internal firm innovation as well as frictional labor mobility and associated idea transmission. In this sense, the paper is close to Akcigit et al. (2016), who also use micro-data to discipline a model of diffusion of ideas, however in the area of patents.

The third strand of the literature examines how worker mobility affects firm outcomes, often by focusing on specific categories of employees—such as engineers (Harrigan et al., 2023), managers and CEOs (Bennedsen et al., 2020), national leaders (Jones and Olken, 2005), inventors (Jaravel et al., 2018)—or by analyzing worker flows more broadly (Stoyanov and Zubanov, 2012). Other studies, such as Parsons and Vézina (2018), highlight the role of immigrants in shaping firms' export behavior. While these prior studies have established that mobile workers can affect firm performance, we advance this literature by isolating the causal effect of the direction of worker mobility—that is, how the characteristics of the sending firm influence outcomes in the receiving firm. In contrast to Stoyanov and Zubanov (2012), who examine the impact of hiring from more productive firms, we leverage quasi-experimental variation arising from trade-induced shocks that generate exogenous changes in worker mobility. This identification strategy mitigates endogeneity concerns related to sorting or selection into specific hiring matches.

The paper proceeds as follows. First, we describe our theoretical framework in Section 2. In Section 3 we outline our data sources, Section 4 shows suggestive correlations while Section 5 provides quasi-experimental empirical results on the importance of worker mobility for firm growth in Sweden. Motivated by the presented evidence, Section 6 shows results for the decomposition of aggregate economic growth and counterfactual exercises using our theoretical framework. Section 7 concludes.

2 Theory

We develop a framework to quantify how worker mobility shapes average firm productivity growth. The theory describes a multi-worker-firm as a learning and innovation environment where productivity change and knowledge diffusion originate. In the model, firm productivity changes either due to worker mobility across firms or due to in-house innovation.

We first lay out the details of the economy (cf. Section 2.1) and describe the surplus function (cf. Section 2.2). We then derive the law of motion for the firm productivity distribution and express average productivity growth (cf. Section 2.3). Section 2.4 presents two extensions used in the empirical analysis, and Section 2.5 summarizes the model's testable predictions. All derivations are relegated to Appendix B.

2.1 Environment

Physical Environment The economy is set in continuous time and there is no aggregate uncertainty. The labor market consists of a mass N_t^I of workers and a mass N_t^J of firms. We index a worker by i and a firm by j. Workers supply one unit of time inelastically to the labor market. Both firms and workers are risk-neutral, infinitely lived, and discount the future at the risk-free rate r. They have heterogeneous and time-varying productivity, denoted p_{it} for workers and P_{jt} for firms. Firm productivity follows an endogenous distribution function $G(P,t)$ with density $g(P,t)$ and survival function $\bar{G}(P,t)$. Firms produce a single homogeneous good with output

$$Y_{jt} \;=\; P_{jt}\,N_{jt}$$

Demographics and entry. Through the inflow of new labor-market participants, the worker mass evolves at rate μ^I; entrant skills are drawn from $W(p,t)$ with density $w(p,t)$. New firms enter at net rate μ^J and draw productivity from the distribution $B(P,t)$ with density $b(P,t)$.

Worker status and search. Workers can be employed or unemployed and there is labor search for both employed and unemployed workers. Let u_t denote the unemployment rate and let the productivity distribution of unemployed workers be $F(p,t)$ with density $f(p,t)$ and survival function $\bar{F}(p,t)$. Each period, the firm exogenously separates from a worker at a Poisson rate δ. Each firm receives a job contact at Poisson rate λ.[2] Conditional on a contact, with probability s the firm meets another firm, and with probability $(1-s)$ it meets the unemployment agency. In a poaching contact, the destination firm is drawn uniformly from the population of firms $G(P,t)$, and one of its workers is selected uniformly as the candidate. In a contact with the unemployment agency, a worker is drawn from the unemployment pool according to $F(p,t)$ subject to the unemployment–agency placement efficiency $\chi(u_t) \in [0,1]$, which we assume weakly increasing in u.

Each job contact gives rise to a potential mobility with associated worker mobility

[2]Due to our interest on firm productivity, we set arrival rates at the firm level rather than the worker level. This allows for considerable gains in tractability and notably sidesteps the need to keep track of a bi-variate distribution in firm productivity and size.

value ξ, following Lamadon et al. (2024). The mobility value is observed at the offer, non-pledgeable, and realized only if the worker moves. It is common knowledge to both workers and firms. We assume that the mobility value is proportional to a reservation value R such that $\xi = \theta R$, with θ drawn from a distribution. For unemployed workers, we assume that the reservation value is the value of unemployment, while for employed workers it is the surplus value of the job, which we specify below.[3] For employed workers, the mobility value is drawn from distribution $\Gamma(\theta)$ with survival function $\bar{\Gamma}(\theta)$; for unemployed workers, it is drawn from $\Gamma_u(\theta_u)$ with survival function $\bar{\Gamma}_u(\theta_u)$.

Firm and worker productivity. Firm productivity P_{jt} evolves through innovation and mobility. Whenever the firm meets a candidate technology ρ—from an unemployed hire with skill p, from an employed origin with productivity P_o, or from an innovation idea draw P'—it upgrades to $\max\{\rho, P_{jt}\}$. The firm innovates by drawing P' from the idea distribution $V(\cdot, t)$ with survival function $\bar{V}(\cdot, t)$ at Poisson rate β.[4] Innovation and hiring can only weakly increase P_{jt}. Worker skills adjust instantaneously to the firm's technology: $p_{it} \leftarrow \max\{p_{it}, P_{jt}\}$.[5]

Contracts, negotiations and mobility. We adopt the continuous–time sequential–auctions protocol of Postel-Vinay and Robin (2002) and Bilal et al. (2022), and with mobility values as in Lamadon et al. (2024). Specifically, offers arrive at Poisson times. At any offer arrival τ, bids and counteroffers depend only on the pre–arrival state and the realized mobility value ξ. Mobility values are non-pledgeable and firm productivity is not verifiable at the offer. If the offer is accepted, the match is formed and any firm–wide learning occurs after acceptance and is not contractible at τ. With counteroffers, the contract equals the worker's second–best valuation.

Acceptance and mobility values. Let $U(t)$ denote the value of unemployment and $S(P, N, t)$ the worker–firm surplus at size N and productivity P. We conjecture (and verify below) that S is affine in firm size N and write $\hat{S}(P, t)$ for the marginal surplus.[6] An employed worker at P who receives an outside offer P' and a mobility value $\xi = \theta\, \hat{S}(P, t)$ accepts if and

[3]This ensures the mobility value never exceeds the total value of the current job.

[4]For any cdf H_t, we use the weak tail (survival) measure $\bar{H}(x, t) := \int_{y \geq x} dH_t(y)$ with $\bar{H}(x, t) = 1 - H_t(x)$.

[5]We follow the literature on knowledge diffusion (Lucas and Moll, 2014) in assuming that knowledge is perfectly transferable across job matches. We study this question empirically in the companion paper Akerman and Holzheu (2025). Using exogenous worker deaths and observed experience at previous firms, we show that a large share of productivity losses due to worker departures can be attributed to transferable productivity.

[6]This guess is further suggested by Bilal et al. (2021) where a linear production setting features this equilibrium structure.

only if $\hat{S}(P',t)+\xi \geq \hat{S}(P,t)$. With $\theta \sim \Gamma(\cdot)$ and survival $\bar{\Gamma}$, the acceptance probability is

$$\Psi(P',P,t)=\bar{\Gamma}\Big(1-\tfrac{\hat{S}(P',t)}{\hat{S}(P,t)}\Big).$$

If the offer is accepted, second-price bidding implies the promise at the destination adjusts so that the worker's total value remains unchanged; if rejected, the incumbent promise updates to the best enforceable inside option. For an unemployed worker with $\xi^{\mathrm{UE}}=\theta_u U(t)$, the offer P' is accepted if and only if $\hat{S}(P',t)+\xi^{\mathrm{UE}} \geq 0$, yielding $\Psi^{\mathrm{UE}}(P',t)=\bar{\Gamma}_u\big(-\hat{S}(P',t)/U(t)\big)$. Details are in Appendix B.2.1. Note that this mechanism differs from Postel-Vinay and Robin (2002): with mobility values, workers may accept offers from lower-productivity destinations if ξ compensates, enabling knowledge flows from higher- to lower-productivity firms outside of hires from unemployment.

2.2 Surplus Function

We write the total value equation of the worker–firm coalition by combining worker and firm value functions (derivations in Appendix B.2.1). Using the joint value, the worker-firm coalition surplus satisfies

$$\begin{aligned}
rS(P,N,t) \quad &= \quad N\,(P-b^u(t))+\dot{S}(P,N,t)\\
&+ \quad \delta\,(\,-\Delta_N S(P,N,t)\,)\\
&+ \quad \lambda(1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\left(\int_P^\infty S_{P',P}(N,t)\,dF(P',t)+\Delta_N S(P,N,t)\right)\\
&+ \quad \lambda s\int_P^\infty \Psi(P,P',t)\;S_{P',P}(N,t)\,dG(P',t)\\
&+ \quad \lambda s\int_{\underline{P}}^\infty \Psi(P,P',t)(\hat{S}(P,t)-\hat{S}(P',t))\,dG(P',t)\\
&+ \quad \beta\int_P^\infty S_{P',P}(N,t)\,dV(P',t)
\end{aligned} \tag{1}$$

where

$$\Delta_N S(P,N,t):=S(P,N,t)-S(P,N-1,t), \qquad S_{P',P}(N,t):=S(P',N,t)-S(P,N,t).$$

The coalition enjoys current profits net of unemployment value and option values from expansion, mobility, and innovation. Relative to a setting without on-the-job search ($s=0$), two EE terms appear: the expected gain from knowledge diffusion when the origin is more productive ($P'>P$) and the gain from the worker's arrival, the marginal surplus, which applies to any accepted EE move. Note that due to the bargaining protocol, workers receive the marginal surplus at the receiving firm, such that no change in expected value is observed when a worker leaves a firm, as in Postel-Vinay and Robin (2002). We verify our guess that this surplus equation permits an affine representation in Appendix B.3. Firm size thus affects the worker–firm surplus only linearly via the slope term, while the intercept is independent of firm size N.

Result 1 (Affine Surplus Representation) *The surplus equation is affine in firm size:* $S(P, N, t) = N\,\hat{S}(P, t) + S^0(P, t)$.

The equilibrium equations for $\hat{S}(P, t)$ and $S^0(P, t)$ are derived in Appendix B.3. In a discrete approximation with grid $\underline{P} = P_1 < \cdots < P_N = \bar{P}$, the system of equations $\hat{S}(P, t)$ is lower triangular in $\{\hat{S}(P_i)\}_{i=1}^N$ and hence admits a unique solution obtained by backward substitution. The solution is strictly increasing in firm productivity P. Intuitively, this derives from the fact that any option value is at most as high as the difference to the next best firm. Proof and regularity conditions are provided in Appendix B.4.

We next study how dispersion in mobility values shapes the direction of job-to-job mobility. Let the mobility value be proportional to the origin surplus, $\xi = \theta\,\hat{S}(P, t)$, where θ is drawn from a scale family $\Gamma_\sigma(\theta) = \Gamma(\theta/\sigma)$ with dispersion $\sigma > 0$ and Γ differentiable. Given an outside offer P' to a worker at P, the acceptance probability can be written as

$$\Psi_\sigma(P', P, t) = \overline{\Gamma}\Big(\tfrac{\tau(P',P,t;\sigma)}{\sigma}\Big), \qquad \tau(P', P, t; \sigma) := 1 - \frac{\hat{S}(P', t; \sigma)}{\hat{S}(P, t; \sigma)}.$$

Dispersion affects acceptance through a direct scale effect on the mobility-value distribution and an indirect effect via the induced change in the surplus ratio. With density $\gamma = \Gamma'$, the derivative is

$$\frac{d\Psi_\sigma(P', P, t)}{d\sigma} = \frac{\gamma(\tau/\sigma)}{\sigma^2}\Big(\tau(P', P, t; \sigma) - \sigma\,\frac{\partial \tau(P', P, t; \sigma)}{\partial \sigma}\Big), \qquad \tau := 1 - \frac{\hat{S}(P', t; \sigma)}{\hat{S}(P, t; \sigma)}.$$

We show in Appendix B.4 (Results 4–5) that $\partial_\sigma \tau(P', P, t; \sigma) \geq 0$, so the indirect term $-\sigma\,\partial_\sigma \tau$ is weakly negative. Hence the direct effect pushes acceptance up; the endogenous response of the surplus profile attenuates it. In particular, starting from low dispersion, greater dispersion raises acceptance of downward offers. For larger σ, the indirect term can dominate and reverse the sign. Increasing dispersion makes large idiosyncratic shocks more likely through the direct effect, which raises acceptance of downward offers; at the same time it differentially raises high-P surpluses relative to low-P surpluses through the indirect effect, tightening the acceptance threshold and counteracting the direct force. Formal proofs and regularity conditions are provided in Appendix B.4.

2.3 Firm Productivity Distribution and Mean Growth

We track the distributions of firm productivity $g(P,t)$ and of the skill of unemployed workers $f(p,t)$. Combining inflows from innovation, entry, and meetings with the corresponding outflows yields the laws of motion for firm productivity and worker productivity as follows

$$\begin{aligned}\dot{g}(P,t) &= \beta\, v(P,t)\, G(P,t) - \beta\, g(P,t)\, \bar{V}(P,t) \\ &\quad + \mu^J\big[b(P,t) - g(P,t)\big] \\ &\quad + \lambda s\, g(P,t)\Big[I^{\downarrow}_{EE}(P,t) - I^{\uparrow}_{EE}(P,t)\Big] \\ &\quad + \lambda(1-s)\chi(u_t)\Big[f(P,t)\, I^{\downarrow}_{UE}(P,t) - \Psi^{\mathrm{UE}}(P,t)\, g(P,t)\, \bar{F}(P,t)\Big] \qquad (2)\end{aligned}$$

$$\dot{f}(p,t) = \delta\frac{N^J_t}{u_t N^I_t} g(p,t) + \frac{\mu^I}{u_t}\, w(p,t) - \lambda_u(t)\chi(u_t) f(p,t) I^{\downarrow}_{UE}(\infty,t) \qquad (3)$$

where we define the interaction operators

$$I^{\downarrow}_{EE}(P,t) := \int_0^P \Psi(P',P,t)\, dG(P',t), \quad I^{\uparrow}_{EE}(P,t) := \int_P^\infty \Psi(P,P',t)\, dG(P',t),$$

$$I^{\downarrow}_{UE}(P,t) := \int_0^P \Psi^{\mathrm{UE}}(P',t)\, dG(P',t).$$

Denote average firm productivity $\bar{P}_t := \int P\, g(P,t)\, dP$. Multiplying (2) by P, integrating and dividing by $\bar{P}_t$ gives the equation governing the growth of average firm productivity:

$$\begin{aligned}\frac{d\bar{P}_t}{dt}\frac{1}{\bar{P}_t} &= \frac{\beta\,\mathbb{E}\big[(P'-P)^+\big]}{\bar{P}_t} + \frac{\lambda(1-s)\chi(u_t)\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(P)\,(p-P)^+\big]}{\bar{P}_t} \\ &\quad + \frac{\lambda s\,\mathbb{E}\big[\Psi(P_d,P_o)\,(P_o-P_d)^+\big]}{\bar{P}_t} + \mu^J\left(\frac{\mathbb{E}_\Lambda[P]}{\bar{P}_t} - 1\right).\end{aligned}$$

where $P \sim G_t$, $p \sim F_t$, $P' \sim V_t$, $(P_d, P_o) \sim G_t \times G_t$, independent. Derivations are collected in Appendix B.5. Note that in the no-dispersion benchmark ($\sigma \to 0$, Postel-Vinay and Robin, 2002), the EE contribution to average productivity growth vanishes and $\Psi^{\mathrm{UE}} = 1$. Hence, in this scenario, only U-to-E mobility can impact average productivity growth.

2.4 Model Extensions

To bring our model to the data, we implement two extensions.

Multi-product firms First, we extend our model to allow for multiple products in line with Bernard et al. (2010) (cf. Appendix B.6). This extension allows us to measure firm productivity both through output per worker as well as through the number of products produced at a firm, with more productive firms producing a larger number of different products. We refer to these two dimensions as vertical productivity growth and horizontal productivity growth, respectively.

Imperfect Knowledge Adoption Second, we allow for imperfect adjustment of productivity after a meeting with a superior technology through worker mobility. When a firm meets a candidate technology ρ, it only partially adopts it. Let $\phi_E \in (0, 1]$ denote the adoption rates for new hires. The post–event productivity is

$$P^{+} \;=\; P \;+\; \phi_E\,(\rho - P)^{\cdot}$$

Under constant adoption rates, the mean-growth equation scales multiplicatively, yielding

$$\begin{aligned} g_t := \frac{d\bar{P}_t}{dt}\frac{1}{\bar{P}_t} &= \frac{\beta\mathbb{E}\big[(P' - P)^{+}\big]}{\bar{P}_t} + \frac{\lambda(1-s)\chi(u_t)\phi_E\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(P)\,(p - P)^{+}\big]}{\bar{P}_t} \\ &\quad + \frac{\lambda s\,\phi_E\mathbb{E}\big[\Psi(P_d, P_o)\,(P_o - P_d)^{+}\big]}{\bar{P}_t} + \mu^J\left(\frac{\mathbb{E}_\Lambda[P]}{\bar{P}_t} - 1\right). \end{aligned}$$

2.5 Summary: Mobility and Growth Patterns

We summarize the mechanisms and predictions of the model. Worker mobility acts as an amplification channel for R&D, transmitting ideas across firms and raising average aggregate productivity. The model yields three predictions.

Prediction 1 (Extent of mobility) *Higher worker mobility increases productivity growth.*

Holding the firm productivity distribution fixed, the probability that a firm upgrades via job-to-job or unemployment-to-employment hiring rises with λ; formally, $\partial g_t/\partial\lambda > 0$.

Prediction 2 (Direction of mobility) *Hiring from more productive origins increases productivity growth.*

Let $\pi_{up}^{EE} \equiv \Pr(P' > P \mid \text{EE hire})$ denote the share of EE hires from higher-P origin firms. Then $\partial g_t/\partial\pi_{up}^{EE} > 0$: the direction of mobility matters for growth.

Prediction 3 (Dispersion in match values) *In economies with low dispersion of match-specific mobility values, an increase in the dispersion parameter σ leads to more worker flows from more to less productive firms and is associated with a higher growth rate g_t.*

In this low dispersion regime, we hence expect $\partial g_t / \partial \sigma > 0$.

We evaluate these predictions using sectoral and microeconomic data. The model implies two measures of productivity gains: (i) direct increases in output per worker, Y/L (vertical growth), and (ii) new product adoption that reflects underlying improvements (horizontal growth). We relate these outcomes to the three mobility shifters emphasized above—the extent of mobility (λ), its direction (π_{up}^{EE}) and workers' valuation dispersion (σ).

3 Data

In the following, we outline our data sources in Section 3.1 and discuss our sample in Section 3.2.

3.1 Data Sources

We leverage three main Swedish data sources: a firm level data set, a worker level dataset and a product-level data set. The set of variables used per dataset, together with the period covered, is summarized in Appendix Table C.1.

The firm-level data derives from the database called *Företagsdatabasen* and includes for example value added, total wage sum and other production costs. The dataset is based on information from the Swedish Tax Authority on administrative registries of the firms' balance sheets. This dataset has been used for example in Cederlöf et al. (2024) and Olsson and Tåg (2025).

The worker-level data is called the *Swedish Longitudinal Integrated Database for Health Insurance and Labour Market Studies (LISA)*. It contains information on all Swedish workers in the private sector, and has previously been used for example by Saez et al. (2019) and Balke and Lamadon (2022). It includes information such as income, education, and age. Occupations are reported according to the Swedish Standard Classification of Occupations (SSYK).[7] We can link employers and employees using firm identifiers and therefore track workers' experience based on the firm they work at. The reliability and quality of this data is regarded as very high, since it is based on tax reports by firms and misreporting is punishable by law.

[7] The base for SSYK is the international standard classification of occupations with reference year 2008 (ISCO-08).

The product-level data is drawn from the dataset *Industrins Varuproduktion (IVP)*, which is based on surveys on the production of Swedish manufacturing firms, and has previously been used by Carlsson and Skans (2012). The dataset includes all firms with at least five employees, and contains information on what products they produce up to the 8-digit Combined Nomenclature (CN) level.[8] For each year, firms report both quantity and price for each product. Using the worker-level data, this product-level data allows us to track workers' experience in specific lines of production.

We also utilize data from the dataset *Research and Development in the private sector* to obtain data on the expenditure of firms on R&D. We specifically use the overall amount in Swedish kronor that firms spend on R&D. The basic criteria for distinguishing R&D from related activities are that there should be an element of innovation and creativity in the activity. The outcome of the activity should be uncertain, and the uncertainty should also apply to the expenditure of financial and human resources. However, the activity should be planned and budgeted and the outcome should be intended to be potentially transferable and replicable in other activities. This dataset has been used for example in Maican et al. (2022) and Arnarson et al. (2024).

To construct the shift–share instruments, we use bilateral product-level trade data from *UN Comtrade*, which provides harmonized statistics on the value and quantity of international trade flows reported by national customs offices. When combined with customs data, these data allow us to measure variation in foreign demand for the goods produced by domestic firms.

3.2 Sample

Our data covers the years 1997–2019. We restrict the population to workers 15–65 years of age with an observed occupation code, and to firms with at least five employees. Table 1 includes summary statistics for our data sample. Our data spans around 1.2 million unique workers working for around 25,000 firms. Workers are on average 43 years old. Around 31% of workers have a college degree and 1% hold a PhD. A worker is likely to leave the current employer with probability 17% each year. Our firm data shows that firms' labor productivity grows at around 2% per year on average. Moreover, the median firm is fairly small and employs around 12 workers. This means that entries of specific workers are likely to be fairly salient for these firms' operations. Around 7% of firms adopt a new product every year, and

[8]The CN system is the EU classification scheme for products, and is used by custom offices as well as statistical agencies, similar to the US equivalent Harmonised System (HS).

around 2% report positive R&D investments every year. This investment intensity is a small number, especially compared to the 17% of workers changing employment each period.

Table 1: Summary Statistics

	Obs. (1)	Mean (2)	SD. (3)	Median (4)	Min. (5)	Max. (6)
A) Workers						
Age	8,105,124	42.91	11.62	43.00	16.00	64.00
Female	8,105,124	0.24	0.43	0.00	0.00	1.00
Income	8,105,124	3,556.81	2,054.78	3,262.00	1.00	437,802.00
Less than HS	8,105,124	0.69	0.46	1.00	0.00	1.00
More than HS	8,105,124	0.31	0.46	0.00	0.00	1.00
Phd	8,105,124	0.01	0.08	0.00	0.00	1.00
Stayer	7,643,679	0.83	0.37	1.00	0.00	1.00
# Workers	1,168,308					
# Firms	21,508					
# Occ. Groups (4-Digits)	366					
B) Firms						
Y/L Growth Rate	212,932	0.02	0.34	0.03	–8.95	7.71
Firm Size	242,380	50.67	329.63	12.00	5.00	22,610.00
New Product Adoption	72,454	0.07	0.25	0.00	0.00	1.00
R&D dummy	242,380	0.02	0.15	0.00	0.00	1.00
# Firms	25,216					
# Prod. Codes (4-Digits)	236					

4 Suggestive Data Patterns

To obtain suggestive evidence for the mechanism in our framework, we assess its Predictions 1–3 across 4-digit sectors. We consider that sectors can potentially vary in key characteristics of labor markets and firm productivity distributions, justifying a cross-sectoral perspective.

Measurement. We relate sector-level productivity growth to two features of worker mobility: (i) its extent - the mean job-to-job mobility rate - and (ii) its direction, that is the share of firms hiring from a more-productive origin. We consider two sector-level proxies for productivity growth: vertical growth as mean growth in labor productivity, Y/L, and horizontal growth, that is the likelihood of new 8-digit product adoption. To probe heterogeneity in knowledge transmission, we compute mobility measures by skill group using occupation–income ranks: top-decile occupations and below-median occupations.

This broader notion of skill complements prior work focusing on technologists, IT workers, inventors, entrepreneurs, and managers (Mion and Opromolla, 2014; Tambe and Hitt, 2014; Jaravel et al., 2018; Becker and Hvide, 2021; Harrigan et al., 2023). In addition, we construct a proxy for the dispersion in workers' valuation of job moves/ job amenities. Workers are first partitioned into groups defined by all possible combinations of four age categories (below 30, 30–40, 40–50, and above 50), four education categories (high school or less, less than two years of college, three to five years of college, and postgraduate), and gender. Sector-level Herfindahl–Hirschman indices (HHI) are calculated using these groups, focusing on the worker distribution among them. The HHI is determined by summing the squares of group shares, meaning a lower index value signifies a more diversified worker composition.

Results. Figure 1 displays the sectoral correlation between mobility and vertical growth. Circles are sized by sector value added; blue denotes below-median occupations and red the top decile. EE rates are positively associated with Y/L growth for high-wage (top-decile) occupations, with a weaker relationship for below-median occupations. The right panel shows similar patterns for the direction of mobility: sectors that hire more often from higher-productivity origins exhibit faster labor-productivity growth. Figure 2 shows analogous results for horizontal product expansion. Taken together, both figures provide suggestive evidence for Predictions 1–2 of the model, that is that the extent and direction of worker mobility correlate with growth. Online Appendix Tables D.1 and D.2 report sector-level regressions underlying the figures, including controls for sectoral labor productivity and volatility.[9] Figure 3 displays the correlation between EE rates in the top decile and growth for industries with high (red color) and low (blue color) dispersion of the amenity valuation proxy, for both vertical and horizontal productivity measures. We find that mobility affects growth more in industries in which worker characteristics are more dispersed, consistent with Prediction 3 of the model: dispersion in amenity valuation is positively correlated with productivity growth, assuming that the direct effect dominates over the indirect effect.

Importance of mobility. To benchmark the correlations of worker mobility and aggregate growth, we run comparative regressions of sectoral productivity growth on standardized mobility measures and a standardized indicator for R&D activity (the sectoral share of R&D-performing firms), using the same specifications as in the Online Appendix Tables D.1

[9] Appendix Tables D.1 and D.2 estimate sector-level regressions separately for bottom-half and top-decile occupations. For Y/L growth (Table D.1), coefficients are consistently larger for the top decile, with differences significant in most cases; product-expansion results (Table D.2) are analogous. Adding controls in columns (3)–(6)—sectoral labor productivity, sectoral volatility, and foreign-demand volatility (the latter via a shift–share measure based on baseline export patterns and global export changes; Hummels et al., 2014)—leaves these patterns intact.

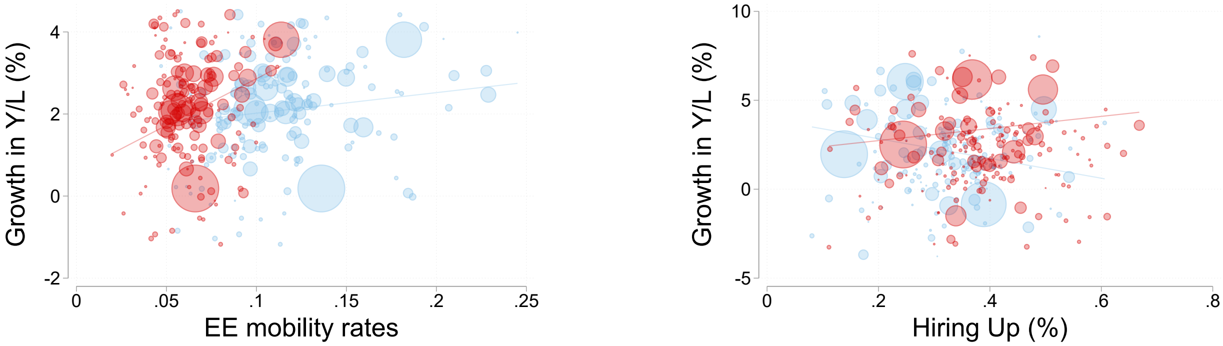

Figure 1: Vertical productivity growth: Growth in Y/L

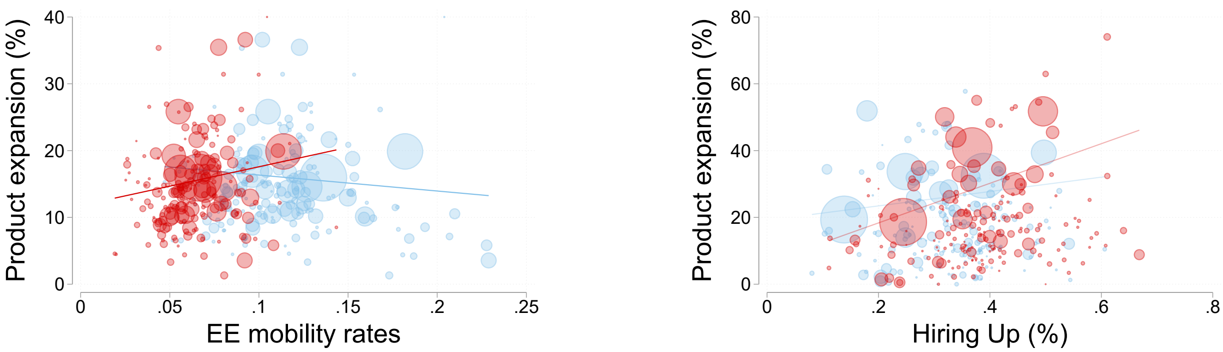

Figure 2: Horizontal productivity growth: Product expansion

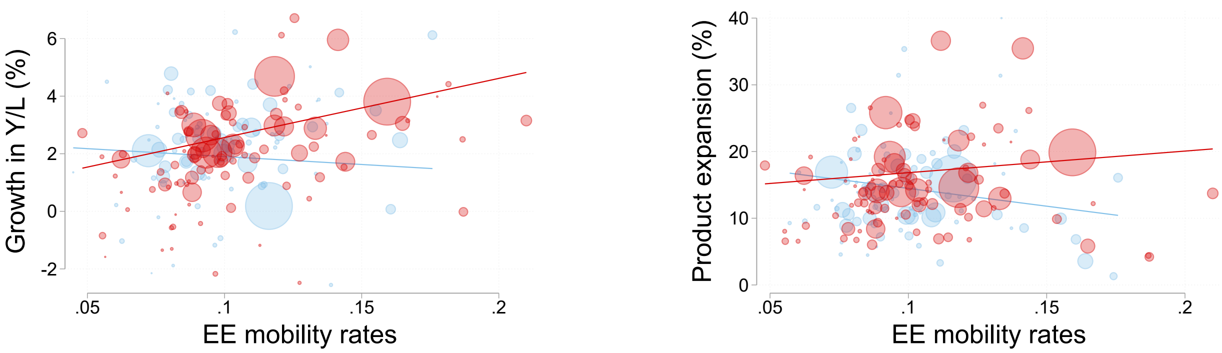

Figure 3: Dispersion in valuation and growth

Notes: Upper panel— Left: binned scatter of EE rates vs. growth in Y/L for below-median (blue) and top-decile (red) occupations. Right: share hiring from higher-productivity origins vs. growth in Y/L. *Middle panel*— Left: EE rates vs. probability of product expansion (blue: all workers; red: top decile). Right: share hiring from higher-productivity origins vs. product expansion. Firm rankings use Y/L in the year prior to mobility. Unit: 4-digit industry, circle size $\propto$ total sales. *Lower panel*— Left: binned scatter of EE rates vs. growth in Y/L for 4-digit industries with more/less (red/blue) dispersion of the amenity-valuation proxy, as defined by whether the sector-specific HHI is below/above the median HHI across sectors. Right: binned scatter of EE rates vs. product expansion, Sweden, 1997–2019.

and D.2. Table 2 shows that, in most specifications, coefficients on job-to-job mobility rates are comparable in magnitude to R&D for top-decile occupations, whereas for below-median occupations R&D typically plays a larger role.

Table 2: Vertical and horizontal productivity growth vs. standardized mobility and R&D

Dep. var.:	Growth in labor productivity		Product expansion	
	Bottom half	Top dec.	Bottom half	Top dec.
	(1)	(2)	(3)	(4)
Panel A				
EE mobility rate	0.003**	0.007***	-0.004	0.013*
(standardized)	(0.001)	(0.001)	(0.005)	(0.004)
R&D activity	0.003***	0.003***	0.016***	0.017***
(standardized)	(0.001)	(0.001)	(0.003)	(0.003)
Observations	191	191	190	190
Panel B				
Hiring up	-0.005***	0.006***	0.044***	0.083***
(standardized)	(0.002)	(0.002)	(0.008)	(0.010)
R&D activity	0.000	0.008***	0.043***	0.070***
(standardized)	(0.002)	(0.002)	(0.009)	(0.009)
Observations	183	180	181	178

Notes: Sector-level regressions of changes in productivity on worker-mobility patterns and R&D activity; coefficients standardized. *** indicates significance at the 1 percent level, ** at the 5 percent level, and * at the 10 percent level.

Job-to-job moves vs. unemployment. If mobility raises productivity through knowledge carried by workers, job-to-job moves could possibly matter more than moves with intervening unemployment. Consistent with this, Online Appendix Figures D.1 and D.3 show strong relationships for EE mobility, whereas Appendix Figures D.2 and D.4 show much weaker relationships when unemployment intervenes.[10]

Taken together, these sectoral patterns—by skill group and by mobility direction—align with Predictions 1–3 of our framework. We next turn to microdata-based designs that deliver causal evidence on the underlying mechanisms.

[10] For separations, EU includes exits into unemployment. For hiring, we classify moves with an unemployment spell between jobs as UE.

5 Quasi-experimental evidence

We use micro-level data to test a central prediction of the model: worker mobility increases firm productivity through knowledge flows. Identification derives from exogenous variation in local hiring opportunities induced by negative trade shocks to a firm's industry x geography peers, following the shift-share design of Hummels et al. (2014). We construct peer shocks by aggregating firm-level trade shocks and disaggregate them by source-firm productivity to study heterogeneity in knowledge transfer by origin firm type. The evidence is consistent with the model (Predictions 1–2): negative shocks to more-productive peers raise hiring locally and yield larger productivity gains at the hiring firm. Our analysis focuses on manufacturing, where export exposure dominates demand conditions, allowing us to use exogenous shifters in labor demand.

5.1 Empirical Strategy

Our identification exploits quasi-random changes in local hiring opportunities created by trade-driven contractions at a firm's peer—firms in the same municipality and sector. When foreign demand falls for what nearby peers sell, those peers shed workers, and local firms face a temporary influx of applicants. We trace the resulting effects on hiring and productivity using a shift–share design in the spirit of Hummels et al. (2014).

Baseline export patterns and variation. We begin from firm-level export data in the base year (2003), disaggregated by HS-6 product and destination. Export scope is narrow: the median product–destination cell is served by only two exporters (seven at the 90th percentile), the median firm serves two countries, and the median firm exports in just three product–destination cells. This concentration generates substantial cross-firm dispersion in exposure to international demand shocks and underpins our design (see Table 3).

Firm-level trade shocks. Using these baseline patterns, we construct firm-specific exposure to subsequent changes in world import demand, Z_{jt}. Let N_c denote destination countries and N_p HS-6 products. For firm j in year t,

$$Z_{jt} \;\; = \;\; \sum_{c \in N_c} \sum_{p \in N_p} s_{jcp} \, M_{cpt},$$

where s_{jcp} is the share of j's 2003 exports accounted for by product p to country c, and M_{cpt} is total global imports of product p by country c in year t. To avoid mechanical links to domestic supply conditions, Swedish exports are excluded when computing M_{cpt}.

Table 3: Export patterns.

	$50th$ Percentile	$90th$ Percentile	$99th$ Percentile	Observations
Country-product-year level				
Number of firms that export within each country-product-year cell	2	7	34	958,774
Firm-year level				
Number of destination country-products	3	43	252	11,645
Number of destination countries	2	18	54	11,645

Notes: Sample includes all firms included to compute shocks to local peers in this section. Firm trade data comes from Swedish Customs, and international trade data is from the UN Comtrade.

Local peer shocks. For each firm i, we aggregate peers' exposure within municipality $m(i)$ and sector $k(i)$ to obtain the local labor-supply shock $\tilde{Z}_{imkt}$:

$$\tilde{Z}_{imkt} = \sum_{j \in J^{-i}_{m(i)k(i)t}} Z_{jt}, \tag{4}$$

where $J^{-i}_{m(i)k(i)t}$ is the set of all firms in $(m(i), k(i))$ at time t excluding i. Negative realizations of $\tilde{Z}_{imkt}$ indicate tightened foreign demand for peers and thus greater worker availability for i.

Event timing and sample window. We implement an event-study design centered on the year in which a firm's peers experience their most severe negative shock; that is, the year t with the largest drop in $\tilde{Z}_{imkt}$ (the firm-specific event year). This captures salient, idiosyncratic demand contractions even when shocks are continuous rather than binary (cf. Bhuller et al., 2013, 2023). We restrict to a balanced panel spanning three years before and after the event year.

Estimation and controls. All outcomes—peer shocks $\tilde{Z}_{imkt}$, output, employment, and productivity growth—are residualized on municipality fixed effects, sector-by-year fixed effects, and year fixed effects. We also control for the firm's own trade exposure, Z_{it}, to isolate variation coming from differentially shocked peers only.

Selection concerns. A potential worry is non-random separations if peers selectively retain workers on unobservables. To address selection on unobservables, we follow the spirit of Card et al. (2016) and estimate an AKM (Abowd et al., 1999) wage model with worker and firm fixed effects; we then control for the change in the firm-year average worker fixed effect. Including this control leaves our results virtually unchanged, consistent with separations not being driven by shifts in unobserved worker quality.

Mechanism test and scope. To probe knowledge transfer, we partition peer shocks by peers' pre-shock labor productivity (above- vs. below-median) and test whether negative shocks to more-productive peers induce larger hiring responses and productivity gains at the hiring firm. Our focus is on manufacturing, where export exposure dominates local demand, and domestic sales to buyers within the same municipality are limited (Akerman et al., 2013). Consistent with the model, the estimates show that negative shocks to productive peers raise hiring and translate into larger productivity improvements at the receiving firm.

5.2 Results

We begin by documenting firms' responses to own demand contractions. Figure 4 plots residualized Z_{jt} in a seven-year window around the firm-specific event year ($t = 0$). The series shows a pronounced, discrete drop at $t = 0$, indicating that the event timing isolates sharp, plausibly exogenous declines in foreign demand. In the same year, the residualized probability of worker separations jumps and then reverts toward baseline; separations rise by approximately 6.5%. These patterns are consistent with negative trade shocks triggering mobility. We then turn to local peer shocks. Figure 5 traces peers' trade shocks, splitting peers by above- versus below-median labor productivity. The shock dynamics are strikingly similar across the two groups, enabling a clean comparison of subsequent outcomes conditional on exposure to workers from more- versus less-productive peers. Figure 6 plots, by event time, the difference in mean residualized outcomes between firms exposed to shocks hitting more- versus less-productive peers (a rise in the series indicates a larger response for the former). Hiring increases following a positive local labor-supply shock—that is, when peers are negatively hit—and the increase is larger when shocks strike more-productive peers. Productivity growth (annual change in log labor productivity) likewise rises more when workers are released from more-productive firms, in line with the prediction that origin-firm productivity shapes the knowledge embedded in mobile workers. Taken together, this evidence provides causal support for our framework's mechanism. Our evidence also supports the view that negative trade-shocks can yield positive impacts in other parts of the economy.

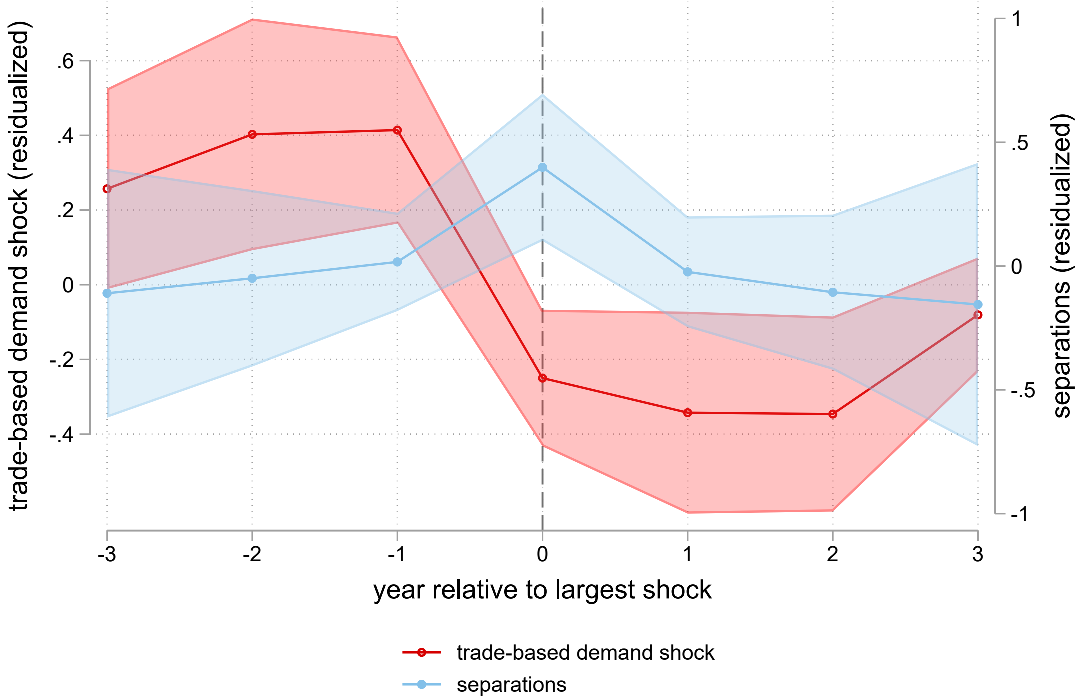

Figure 4: Effect of foreign demand on separations.

Notes: We define each firm's event year as the year with the largest decline in world import demand for its product–destination mix. We keep a balanced ±3-year window around this event, retaining only firms observed in all seven years. Firm-level trade shocks and separations are residualized on municipality fixed effects and sector-by-year fixed effects. We then plot average residualized separations and foreign export demand by event time. Bootstrapped 95 percent confidence intervals are shown. They are constructed from 500 bootstrap replications, each based on a random draw of 95 percent of the sample without replacement.

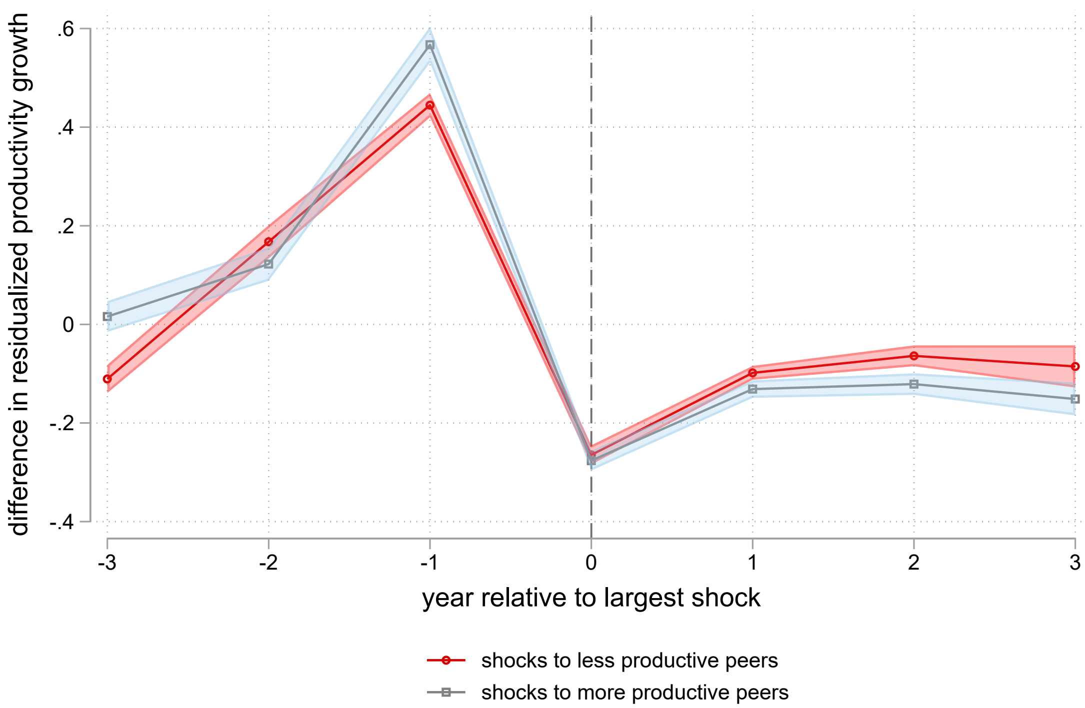

Figure 5: Evolution of shocks to local peers.

Notes: We define each firm's event year as the year with the largest decline in world import demand for its product–destination mix. We keep a balanced ±3-year window around this event, retaining only firms observed in all seven years. Firm-level trade shocks and separations are residualized on municipality fixed effects and sector-by-year fixed effects. The figure then plots total peer trade shocks for peers with above- and below-median productivity, respectively, over event time. Bootstrapped 95 percent confidence intervals are shown. They are constructed from 500 bootstrap replications, each based on a random draw of 95 percent of the sample without replacement.

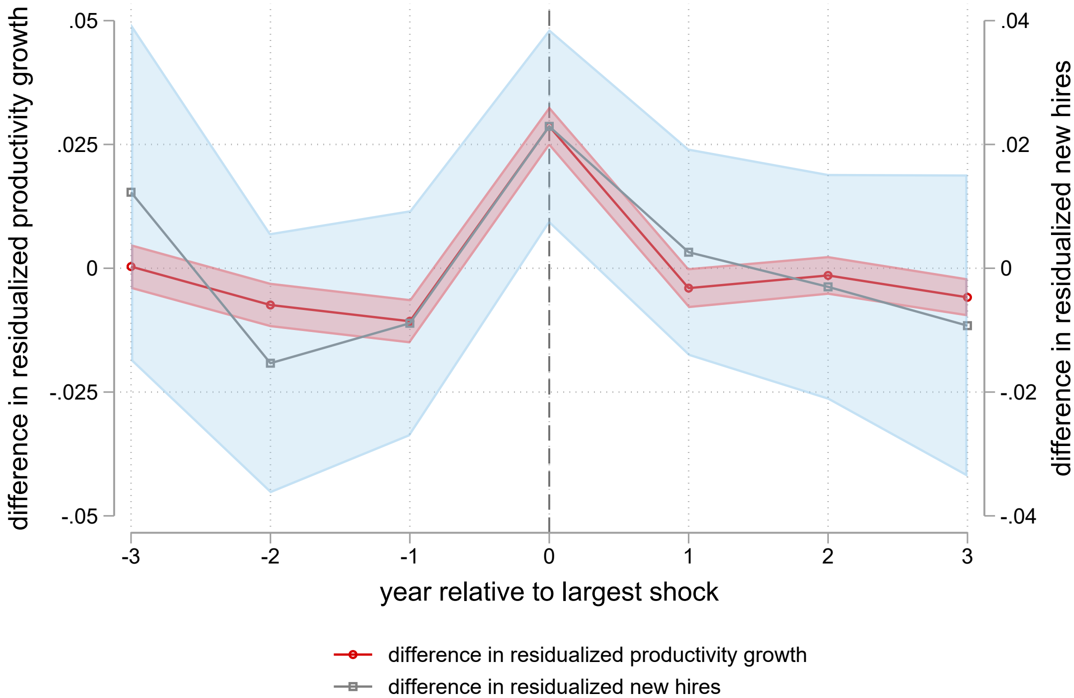

Figure 6: Evolution of hiring patterns and productivity growth during a local labor supply shock.

Notes: We define each firm's event year as the year with the largest decline in world import demand for its product–destination mix. We construct a balanced ±3-year panel around this event, retaining only firms observed in all seven years. Firm-level productivity growth and new hires are residualized on municipality fixed effects and sector-by-year fixed effects. The figure then plots the difference in average residualized outcomes between when the shocks affect high-productivity peers and when the shocks affect low-productivity peers, over event time. A positive value indicates that firms where high-productivity peers are shocked experience larger gains in productivity or hiring following the shock. Bootstrapped 95 percent confidence intervals are shown. They are constructed from 500 bootstrap replications, each based on a random draw of 95 percent of the sample without replacement.

6 Growth Contributions

Motivated by the empirical patterns, we quantify the model's decomposition of productivity growth using firm- and worker-level micro data. We first describe the calibration in Section 6.1, then present the decomposition results in Section 6.2. Section 6.3 presents counterfactual exercises to set our decomposition results in perspective.

6.1 Calibration and Measurement

We begin from the growth-accounting identity (see Section B.5):

$$\frac{\dot{\bar{P}}_t}{\bar{P}_t} = \frac{\beta\,\mathbb{E}[(P'-P)^+]}{\bar{P}_t} + \frac{\lambda(1-s)\chi(u_t)\,\phi_E\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(p)\,(p-P)^+\big]}{\bar{P}_t} + \frac{\lambda s\,\phi_E\,\mathbb{E}[\Psi(P_d,P_o)\,(P_o-P_d)^+]}{\bar{P}_t} + \mu^J\left(\frac{\mathbb{E}_\Lambda[P]}{\bar{P}_t}-1\right). \tag{5}$$

where $x^+ \equiv \max\{x, 0\}$. We calibrate the model-implied objects with micro data and identify growth contributions nonparametrically. In the following, we detail how each expectation (and its associated acceptance term) is measured, and how β and ϕ_E are identified. This mapping allows us to decompose (5) term by term. Details are collected in Appendix E.1.

Data and discretization We use firm-year micro data comprising: (i) value added and employment to construct $P_{jt} \equiv Y_{jt}/N_{jt}$; (ii) accepted job-to-job moves with origin and destination firm identifiers; (iii) hires from unemployment at the firm-year level; (iv) R&D outlays as a proxy for innovation; and (v) demand shifters measured via shift-share instruments to absorb revenue-side noise. For transparent implementation, we discretize productivity into bins $\mathcal{P} = \{P^1, \ldots, P^K\}$. In practice, we set $K = 20$. Let $G_t(k)$ denote the share of firms in bin k (consistent with firm-level offer arrival in the model) and $F_t(\ell)$ the share of unemployed workers with skill ℓ. All expectations below admit plug-in estimators that replace integrals with finite-sample sums.

EE acceptance and the EE expectation Let $h_{\mathrm{out}}(o,t)$ denote the per-firm hazard of accepted EE outflow from origin bin o, $S(d \mid o,t)$ the destination share among accepted moves, and $G_t(d)$ the destination-firm share. Under firm-level offer arrival with intensity $c_t \equiv \lambda s$,

$$h_{\mathrm{out}}(o,t) = c_t \int \Psi(d,o,t)\,dG_t(d), \qquad S(d \mid o,t) = \frac{G_t(d)\,\Psi(d,o,t)}{\int G_t(y)\,\Psi(y,o,t)\,dy}.$$

Combining these relations identifies the compound acceptance kernel:

$$\lambda s\,\Psi(d,o,t) \;=\; h_{\text{out}}(o,t)\,\frac{S(d\mid o,t)}{G_t(d)}.$$

Based on the acceptance kernel, together with our estimate for ϕ_E, we compute the EE contribution as the expectation over origin and destination using a plug-in estimator based on the discrete productivity grid.

Calibrating ϕ_E. An event study at destinations around accepted EE arrivals identifies the average adoption fraction:

$$\widehat{\phi}_E \;=\; \frac{\sum_{\text{E events }(o\to d,t)} \Delta P_{d,t\to t+\Delta}\;\cdot\;\mathbf{1}\{\text{no R\&D shock, no UE hire in }[t,t+\Delta]\}}{\sum_{\text{EE events }(o\to d,t)}(P_{o,t^-}-P_{d,t^-})^+},$$

using a one-year window Δ. If multiple EE hires arrive within the window, we collapse to a single event with $(P_o-P_d)^+$ taken from the maximal-origin pair as consistent with the model. By construction, the denominator is observed at the event time from origin–destination pre-move productivities. To address potential endogeneity concerns, we employ both an event-study design and an instrumental variables (IV) strategy based on the peer trade-shock framework described in Section 5 to obtain a causal estimate of the adoption intensity, ϕ_E. We first restrict the sample to the year before and the year of each event, that is, the period when a firm's peers experience their largest labor-supply shock. We then compute the difference between the average productivity of the firm's peers and its own productivity in the pre-event year, $(P_{o,t-1}-P_{d,t-1})$. Finally, we regress the change in firm i's productivity from the pre-event year to the event year, $\Delta P_{i,t\to t+1}$, on this pre-event productivity gap, controlling for two-digit sector and municipality fixed effects (δ_{sy}):

$$\Delta P_{i,t\to t+1} = \phi_E(\overline{P}_{o(i),t-1} - P_{i,t-1}) + \delta_{sy} + \epsilon_{it} \tag{6}$$

Innovation We report two complementary specifications. First, we map observed R&D to innovation arrival in the model, factorizing the term $\beta\,\mathbb{E}[(P'-P)^+]$. Second, we treat innovation as the residual needed to rationalize average positive productivity growth after removing the worker mobility and entry components. These two approaches address potential R&D measurement concerns and hence yield complementary insights. In the R&D-based specification, we estimate firm-level innovation incidence from observed expenditures and observables and aggregate the predicted incidence to obtain $\widehat{\beta}$. We then estimate the reduced-form productivity gain per innovating firm and recover $\mathbb{E}[(P'-P)^+ \mid \text{Innov}=1]$. The product $\widehat{\beta}\times\widehat{\mathbb{E}[(P'-P)^+\mid\text{Innov}=1]}$ pins down the innovation contribution in (5). In the residual specification, we compute the difference between realized productivity growth and the non-innovation terms.

Firm Entry Let Λ_t denote firms that enter between t and $t+1$. Let $\bar{P}_t$ be the incumbent mean at t and $\mathbb{E}_\Lambda[P]$ the mean productivity of entrants in their first observed year. Denote by μ^J the net firm-mass growth rate from entry and exit. The contribution of firm turnover to aggregate productivity growth is

$$\mathcal{C}_t^J \;=\; \mu^J\left(\frac{\mathbb{E}_\Lambda[P]}{\bar{P}_t}-1\right).$$

We measure $\mathbb{E}_\Lambda[P]$ from entrant cohorts and μ^J as net firm-count growth.

UE acceptance and the UE expectation With firm–level arrivals, a firm in destination bin d receives UE offers at rate $\lambda(1-s)\chi(u_t)$; offers are accepted with probability $\Psi^{UE}(d,t)$, which depends only on the destination. Let $h_{\text{in}}^{UE}(d,t)$ denote the per–firm hazard of accepted UE inflows into d (UE hires per firm). Then

$$h_{\text{in}}^{UE}(d,t) = \lambda(1-s)\chi(u_t)\,\Psi^{UE}(d,t). \tag{7}$$

To take expectations over the UE contribution across destinations and origins, we proceed in two ways. First, we take expectations under the steady–state firm distribution $f(p)$, imposing $w(p)=g(p)$. Second, we proxy the firm distribution by that of newly unemployed workers, $\hat{F}$, and approximate $F(\ell)=\hat{F}(\ell)$. We report results under both approaches.

6.2 Parameter Estimates and Growth Decomposition

Parameter estimates Table 5 reports estimates of β and ϕ_E. For ϕ_E, we present both baseline estimates and quasi-experimental estimates from the trade-shock IV specification. Adoption intensities from worker mobility are modest relative to the full-adoption benchmark of 1 with $\hat{\phi}_E = 0.13$ for the baseline and $\hat{\phi}_E = 0.21$ for the quasi-experimental estimate.[11] Figure 7 illustrates the estimation of ϕ in the latter case. Each point represents an event, and observations are grouped into 20 quantiles based on the pre-event productivity gap, with an equal number of observations per quantile. The figure shows that when the productivity of peers exceeds that of the firm (a positive gap), productivity growth is increasing in the

[11] To address concerns that adoption rates may differ between E–E and U–E moves, we estimate ϕ_E in an event–study around U–E transitions using recently unemployed workers. The estimate, $\hat{\phi}_E = 0.128$, is closely aligned with our baseline. We therefore find no evidence that differential adoption rates cause us to overstate the U–E contribution.

Table 5: Parameter Estimates

	β	ϕ_E	
		Baseline	Trade Shock
Estimate	0.026	0.127	0.206

gap—consistent with learning from more productive peers. By contrast, when the firm is more productive than its peers (a negative gap), differences in the gap have no discernible effect on productivity growth, consistent with the model's assumptions. The innovation arrival rate is

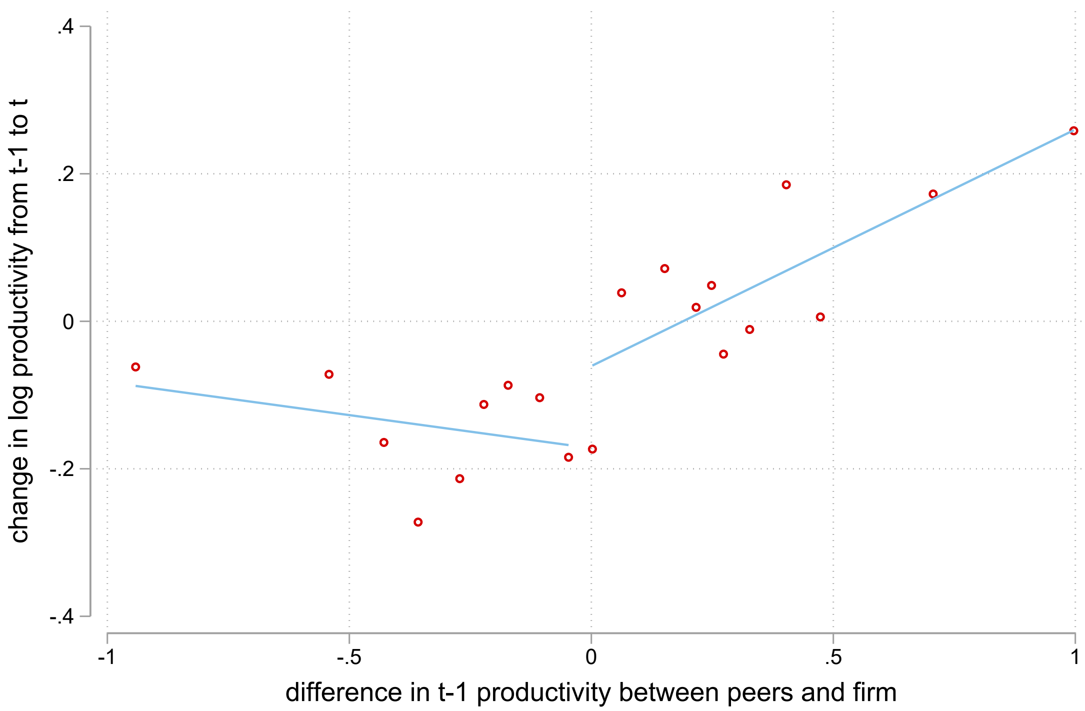

Figure 7: Change in Firm Productivity and the Initial Productivity Gap with Peers.

Notes: The figure illustrates the regression in equation 6. It focuses on events where firms' peers experience large shocks to foreign demand for their exports, as described in Section 5. The vertical axis shows the change in log productivity from the year before the event to the event year. The horizontal axis measures the pre-event productivity gap between the firm's peers and the firm. Each observation corresponds to an event, and observations are grouped into 20 quantiles, each with the same number of observations, along the horizontal axis.

$\hat{\beta} = 0.026$, implying that a small share of firms innovate in a given year after conditioning on observables, consistent with evidence that most firms report zero R&D expenditure (Klette and Kortum, 2004).

Growth Decomposition Using the estimated parameters, we decompose positive aggregate labor-productivity growth into four components: (i) innovation; (ii) job-to-job mobility; (iii) unemployment-to-employment mobility; and (iv) firm entry. As our model rationalizes growth, we focus our analysis on firms with positive growth experience. Table 6 reports results for six specifications. Columns (1)–(3) use the baseline estimate of ϕ_E from aggregate mobility rates; columns (4)–(6) use the trade-shock IV estimate. Columns (1) and (5) treat innovation as observed in the R&D data ("Observed"), whereas the other columns attribute the residual of average positive productivity growth to innovation after removing mobility and entry ("Back-out"), thereby addressing potential mismeasurement of innovative activity. Within the back-out columns (2) and (3) and (5) and (6), we consider two versions for the origin distribution of UE mobility events. The results show that the model explains between 54%

Table 6: Growth Decomposition

		Adoption Rate ϕ_E					
		Baseline			Trade Shock		
		Innovation Contribution					
		Observed	Back-out		Observed	Back-out	
			Origin Dist. UE			**Origin Dist. UE**	
			$\tilde{F}$	G		$\tilde{F}$	G
Innovation		0.23	47.00	54.00	0.23	16.00	27.00
Mobility		5.62	51.00	44.00	9.01	83.00	72.00
	E-to-E	3.70	33.00	33.00	5.93	54.00	54.00
	U-to-E	1.92	18.00	11.00	3.08	29.00	18.00
Firm Arrival		0.16	1.00	1.00	0.16	1.00	1.00
Total		6.00	100.00	100.00	10.00	100.00	100.00
Average pos. growth		11.0					

Notes: "Observed" treats innovation as measured in the R&D data (levels); "Back-out" infers innovation as the residual share. Total sums the individual components. Columns (1) and(4) are in percentage points, the remainder in shares of average positive growth.

and 91% of average positive growth when using R&D information; by construction, in columns (2),(3) and (5),(6) the model accounts for 100% of positive growth. Across all specifications, mobility is a significant contributor to average positive labor-productivity growth, with EE the largest component within worker mobility. Across specifications, worker mobility accounts for as much as 93% (column 1) or as low as 44% (column 3) of growth. The UE contribution is about one-third to one-half of the EE contribution, reflecting different mobility patterns for workers moving from unemployment versus employment. Recognizing potential uncertainty about the UE magnitude, the "Back-out" specifications provide a conservative lower bound:

at least 33%–54% of aggregate growth is attributable to worker mobility.

6.3 Counterfactual Exercises

6.3.1 Calibrating and Solving the Model

To conduct counterfactual exercises, we calibrate and solve the Bellman equation that determines the slope of the affine surplus function, $\hat{S}(P)$, which governs job-acceptance decisions. Our procedure is reminiscent of Hotz and Miller (1993) in that we use observed mobility flows to uncover value rankings. We start from a steady-state allocation and analyze growth following changes in the model's parameters. We discipline the model using the joint distribution of firm productivity and worker mobility and approximate the job-value shock distributions Γ and Γ_u as logits.
First, imposing stationarity of the firm–productivity distribution and using observed firm and entrant densities together with UE and EE flows by origin–destination productivity, we identify the innovation intensity on a fine productivity grid. We then aggregate the grid into low- and high-productivity bins. On this grid, UE flow ratios identify the relative surplus slopes, EE flow ratios identify the taste parameter in employed workers' acceptance decisions, and the levels of total UE and EE movers pin down offer arrival rates. Finally, conditional on these objects, the Bellman equation for the surplus slope in the low bin pins down the level of the unemployment value, which scales surplus without affecting the previously identified shapes. Details are collected in Appendix E.2.

6.3.2 Counterfactual experiments

We parameterize mobility and selection by the vector of counterfactual levers ξ

$$\theta = (\xi^{\lambda}, \xi^{\chi}, \xi^{\eta}, \xi^{\sigma}, \xi^{\phi}),$$

where ξ^{λ} scales the overall arrival rate of meetings, ξ^{χ} scales the unemployed job contact index, ξ^{η} tilts the offer distribution toward high–productivity firms, ξ^{σ} denotes the scale shifter in Γ, Γ_u and ξ^{ϕ} denotes the shifter increasing knowledge adoption. The baseline is

$$\theta^0 = (1,\ 1,\ 0,\ 1, 1),$$

where $\xi^{\eta} = 0$ corresponds to no tilt in the offer distribution. For each multiplicative lever, we consider a 10% increase relative to its baseline value, setting $\xi^{\lambda} = \xi^{\chi} = \xi^{\sigma} = \xi^{\phi} = 1.1$ in turn while holding the others at their baseline. For the tilt parameter, we instead consider a positive log-odds shift $\eta > 0$ (corresponding to $\xi^{\eta} > 0$) starting from the baseline $\xi^{\eta} = 0$. The first experiment, with $\xi^{\lambda} = 1.1$, isolates the effect of faster matching at fixed selection by increasing job-finding rates for both types of job arrivals symmetrically. The second experiment, with $\xi^{\chi} = 1.1$, scales the efficiency of the unemployment agency, thereby increasing the arrival rate of UE job contacts. The third experiment introduces an offer-tilt parameter $\eta > 0$ (i.e., $\xi^{\eta} > 0$) that shifts the meeting probability toward high-productivity firms without changing the cross-sectional firm distribution. Let p_H^0 denote the baseline probability that a random meeting draws a high-bin firm, $p_H^0 = g_H/(g_L + g_H)$. In the counterfactual we define the tilted high-offer probability $p_H(\eta)$ by a log-odds shift,

$$\log\left(\frac{p_H(\eta)}{1 - p_H(\eta)}\right) = \log\left(\frac{p_H^0}{1 - p_H^0}\right) + \eta,$$

and use $p_H(\eta)$ to construct effective high-bin meeting weights in the UE and EE blocks. The fourth experiment, with $\xi^{\sigma} = 1.1$, introduces a scale parameter in the mobility-shock distribution σ as discussed in Section 2.2, such that $\Psi_{\sigma}(P', P, t) = \bar{\Gamma}\big(\tau(P', P, t)/\sigma\big)$, and analogously for Γ_u. Finally, the fifth experiment increases the knowledge-adoption rate by 10% (i.e., sets $\xi^{\phi} = 1.1$). These variations correspond to clear policy experiments - the first three address changes in labor market frictions regarding both the extent and the direction of mobility, whereas the last two concern firms' diversity or HR policies.

6.3.3 Counterfactual results

Table 7 reports the counterfactual outcomes alongside the baseline, focusing on the components that vary in the counterfactual exercises, namely the UE and EE contributions. The baseline matches the magnitudes in the reduced-form decomposition (without model calibration) reasonably well, indicating that the structural model can rationalize the observed pattern of growth contributions. Across the counterfactuals, two patterns emerge. First, as expected, a rise in the job-finding rate increases average growth, as shown in the first and second counterfactual exercises. Second, changes that alter the composition of mobility—either by tilting offers toward high-productivity firms or by increasing the dispersion of mobility shocks—have large effects on average growth. Similarly, improvements in the adoption capacity ϕ significantly increase average growth. Taken together, these results show that changes in the direction of mobility can be as important for growth as changes in its overall level.

To set these results in perspective, consider a change in EPL legislation. Estimates from OECD (2010) and Martin and Scarpetta (2011) indicate that a one-point increase in the OECD EPL index on regular employment —equivalent to the difference in the EPL

Table 7: Baseline and Counterfactual Results

Scenario	Policy Lever	UE	EE	Total	Share
Baseline		1.65	3.42	6.35	0.80
CF1: $\xi^{\lambda} = 1.1$	Labor Market Frictions	1.81	3.76	6.86	0.81
$\xi^{\lambda} = 1.2$	EPL Reduction (level Australia)	1.97	4.10	7.37	0.83
CF2: $\xi^{\chi} = 1.1$	Labor Market Frictions	1.81	3.42	6.52	0.80
CF3: $\xi^{\eta} = 1$	Targeted Matching	2.30	4.78	8.37	0.85
CF4: $\xi^{\sigma} = 1.1$	Diversity Policies	6.21	3.45	10.95	0.88
CF5: $\xi^{\phi} = 1.1$	HR-Policies/Worker On-boarding	1.81	3.76	6.86	0.81

Notes: The table contains counterfactual changes in positive growth for five counterfactual exercises together with the counterfactual levers and the share of mobility in growth. Results are shown in percentage points.

index between Sweden and Australia in 2010 —reduces job-to-job mobility by approximately 2 percentage points, corresponding to a roughly 20% lower meeting rate.[12] Applying these estimates to our framework implies that a relaxation of Sweden's EPL (to the Australian level) would increase average aggregate growth by 1.02 percentage points, holding other factors constant.

We relate these findings to the empirical evidence in Section 4 via a variance decomposition of labor-productivity growth across sectors (Table 8, Panel A). Cross-sectionally, mobility accounts for most of the variance, with effects especially pronounced when restricting to workers in the top income decile. While the empirical and model objects do not map one-to-one, the patterns reinforce the central role of labor mobility in aggregate productivity growth. In Panel B, we study heterogeneity by managerial practices by sorting sectors into equally sized groups by the share of workers in human-resource (HR) occupations, identified via occupational codes.[13] Sectors with a higher-than-median share of HR workers (the median share across firms is around 1%) exhibit a larger share of the productivity-growth variance explained by mobility, consistent with the view that human-capital management—e.g., more effective onboarding that raises ϕ_E, diversities policies that increase σ or targeted matching that increases η —amplifies the contribution of mobility.

These results invite clear policy conclusions. To raise growth through worker reallocation, the model suggests that policies should primarily increase matching to workers who can

[12]In 2010, Sweden featured an EPL index of 2.61, above the OECD average of 2.15, with Australia featuring an EPL index of 1.67. Source: OECD ELP Index 2010.

[13]We classify HR managers (levels 1 and 2) and personnel/HR specialists (SSYK 1221, 1222, 2423).

Table 8: Share of variation in productivity growth explained by mobility versus innovation.

Dep. var: productivity growth		
	Category of occupations	
Panel A: By occupation charact.	Bottom half	Top ten percent
Factor		
EE mobility	54%	83%
R&D	20%	35%
Covariance	39%	1%
	HR staff	
Panel B: By sector charact.	Less	More
Factor		
EE mobility	4%	34%
R&D	68%	36%
Covariance	29%	30%

Notes: The table contains estimates of a variance composition of residual productivity growth on EE mobility, R&D and the covariance between these two factors. Residualization has been performed with respect to productivity, trade-shocks and firm size. Estimates denote the shares in percent that each factor explains out of total variation explained.

transmit productivity gains and focus on job-to-job mobility among the employed. The counterfactuals also show that the direction of mobility, not just its level, is an important driver of growth, in line with our empirical evidence. Taken together, these results underscore the importance of worker mobility for aggregate growth.

7 Conclusion

We provide both theoretical and empirical evidence that worker mobility amplifies firm growth. First, we develop a random search model that rationalizes worker mobility from high to low quality firms, both for UE and EE transitions. Second, using highly granular data on workers and firms in Sweden, we document aggregate and microeconomic evidence for a link between worker mobility and labor productivity growth. In a calibration exercise at the aggregate level, we find evidence that both worker mobility and R&D shape average firm productivity growth, with worker mobility from employment taking a significant role. Counterfactual exercises show that the composition of worker mobility significantly affects growth.

Taken together, our results have clear implications for productivity growth in light of the secular decline in worker mobility across firms and regions and persistent cross-country gaps. First, holding other factors fixed, lower labor mobility reduces aggregate productivity growth, so institutions that damp mobility—such as employment protection—can depress aggregate productivity independently of their effects on firms' R&D incentives (cf. Aghion et al., 2023). Second, we show that increasing knowledge adoption or raising contact rates of high and low productive firms raises growth. Third, the immediate growth contribution from new firm creation is lower than that from labor mobility, underscoring the importance of new-firm hiring for aggregate growth. Moreover, greater dispersion in workers' mobility valuations directly raises growth, apart from any innovation benefits associated with worker diversity. These mechanisms are particularly relevant in downturns, when displacements increase the share of workers coming from lower-quality firms and compress growth-enhancing mobility flows, further amplifying productivity declines.

Our analysis points to several directions for further research. In our baseline framework, firms' innovation intensity is fixed. Allowing innovation to respond endogenously to worker mobility would permit analysis of firms' incentives when mobility may dilute the private returns to innovation and would provide a natural setting to study contractual mechanisms such as noncompete agreements. While such contracts are salient in the United States, they appear less relevant in Swedish manufacturing. It would also be valuable to introduce decreasing returns with endogenous vacancy posting, though this would come at the cost of several tractable features of the framework. Our preference for analytical simplicity is further motivated by the observation that, if firms expand hiring after innovating—potentially recruiting from more productive firms—innovation would induce additional knowledge diffusion through mobility, likely strengthening the role of mobility documented in Section 6.

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Appendix A Online Appendix

The following online appendix assembles details for the theoretical section of the paper (cf. section B) as well as additional details on the data set (cf. section C), empirical evidence (cf. section D) and estimation results (cf. section E.2).

Appendix B Additional Details for Theoretical Framework

B.1 Notation

Symbol	Meaning
r	Discount rate
λ	Intensity of search
s	Relative intensity of employed search
δ	Firm-level EU separation hazard
β	Innovation arrival rate
ϕ_E	Knowledge Adoption Rate
$F(\cdot, t)$	Distribution of productivities of the unemployed
$G(\cdot, t)$	Distribution of firm productivities
$V(\cdot, t)$	Distribution of innovation ideas
$B(\cdot, t)$	Distribution of new firm entrants
$\Gamma(\cdot)$	Distribution of mobility value shocks
N_t^J	Number of firms at time t
N_t^I	Labor force
θ, θ_u	EE mobility draw, UE mobility draw
P, p	Productivities of firms or workers
N_j	Size of firm j
u_t	Unemployment rate
$\chi(u_t)$	Unemployment Agency Efficiency Rate
$b^u(t)$	Flow unemployment benefit
$\tau(P_{\text{dest}}, P_{\text{orig}}, t)$	Acceptance threshold for an EE offer
$\Psi(P_{\text{dest}}, P_{\text{orig}}, t)$	EE acceptance probability
$\Psi^{UE}(P_{\text{dest}}, t)$	UE acceptance probability
$U(t)$	Value of unemployment
$\mathcal{R}$	Promised pledgeable utility to a worker
$\mathcal{W}^\star(P, N, \mathcal{R}, t)$	Worker value
$w(P, N, \mathcal{R}, t)$	Wage consistent with $\mathcal{W}^\star$
$R(P, \theta, t)$	Second-price pledgeable promise paid when moving from P
$R^U(\theta_u, t)$	Second price against unemployment
$\mathcal{V}(P, t)$	Worker's total continuation value upon accepting EE offer w/ origin P
$\mathcal{R}^+(P_{\text{dest}}, P_{\text{orig}}, \theta, \mathcal{R}, t)$	Promise update at the incumbent after a rejected EE offer P_{dest}
$\boldsymbol{\mathcal{R}}$	Vector of workers' promises $(\mathcal{R}_1, \ldots, \mathcal{R}_N)$
$\boldsymbol{\mathcal{R}}^{-k}$	Promise vector with entry k removed
$\boldsymbol{\mathcal{R}} \oplus \{x\}$	Promise vector with x appended
$\boldsymbol{\mathcal{R}}^{(k)\uparrow}(P'', \theta)$	Promise vector after worker k rejects to P'': replace $\mathcal{R}_k$ by $\mathcal{R}^+(P'', P, \theta, \mathcal{R}_k, t)$
$\mathcal{W}^\Sigma(P, N, \boldsymbol{\mathcal{R}}, t)$	Total payroll
$J(P, N, \boldsymbol{\mathcal{R}}, t)$	Firm value
$\Omega(P, N, \boldsymbol{\mathcal{R}}, t)$	Coalition value
$S(P, N, t)$	Coalition surplus: $S = \Omega - N\,U(t)$
$\Delta_N S(P, N, t)$	Size difference: $S(P, N, t) - S(P, N-1, t)$
$S_{P', P}(N, t)$	Productivity jump difference: $S(P', N, t) - S(P, N, t)$
$\hat{S}(P, t)$	Marginal surplus of adding one worker at productivity P

B.2 Model Derivations - HJB Equations

B.2.1 Worker offers, acceptance and values

Unemployment value. A worker in unemployment receives value $U(t)$. It is composed of the flow value of receiving unemployment benefits $b^u(t)$ and the option value of matching with a firm. Given the negotiation protocol, the worker is offered his second best outcome when hired from unemployment, leaving his total value before and after job finding unchanged. Hence the value of unemployment $U(t)$ satisfies

$$r\,U(t) = b^u(t) + \dot{U}(t).$$

Acceptance rule and second price after job offer. A worker employed at a firm with productivity P has state $(P, N, \mathcal{R}, t)$, where $\mathcal{R}$ is the promised pledgeable utility. At an outside offer from firm P' to a worker employed at P, the worker draws a shock θ with

$$\xi = \theta\,\hat{S}(P,t), \qquad \theta \in [-\underline{\theta}, \overline{\theta}], \quad \overline{\theta} \leq 1,$$

independent of productivities (P, P') with CDF Γ and survival function $\bar{\Gamma}$. The mobility shock is proportional to the incumbent surplus and non-pledgeable. It is observed at the time of the offer and realized only if the worker accepts the new job. Workers never move to unemployment after receiving an outside offer, since remaining with the incumbent guarantees a surplus at least as high as unemployment.

Job acceptance then proceeds as follows. At the outside offer P', the worker compares the destination marginal surplus $\hat{S}(P',t)$ plus the mobility value ξ to the incumbent offer at the current job, $\hat{S}(P,t)$. Acceptance is deterministic given (P, P', ξ, t):

$$\text{accept}(P, P', \xi, t) \iff \hat{S}(P',t) + \xi \; > \hat{S}(P,t).$$

Similarly,

$$\text{accept}(P, P', \theta, t) \iff \quad \theta \; > \tau(P', P, t) := 1 - \frac{\hat{S}(P',t)}{\hat{S}(P,t)}.$$

We can hence define the acceptance probability

$$\Psi(P', P, t) \;:=\; \Pr\Big(\theta > \tau(P', P, t)\Big) \;=\; \bar{\Gamma}\Big(1 - \tfrac{\hat{S}(P',t)}{\hat{S}(P,t)}\Big).$$

If accepted, the second-price promise at the destination, their promised pledgeable utility, equals

$$\mathcal{R}(P, \theta, t) = U(t) + \hat{S}(P, t)\,(1 - \theta),$$

so the worker's total continuation value at the destination is

$$\mathcal{V}(P, t) = \mathcal{R}(P, \theta, t) + \xi \;=\; U(t) + \hat{S}(P, t),$$

i.e., equal to the incumbent total. If rejected, the incumbent promise updates to the best enforceable inside option:

$$\mathcal{R}^{+}(P', P, \theta, \mathcal{R}, t) = \max\bigl\{\mathcal{R},\; U(t) + \hat{S}(P', t) + \theta\,\hat{S}(P, t)\bigr\}.$$

Similarly, when an unemployed worker receives a job offer P' and a mobility shock ξ, they will accept the job offer if

$$\text{accept}^{\text{UE}}(P', \xi, t) \iff \hat{S}(P', t) + \xi^{UE} \;\geq\; 0 \qquad \xi^{\text{UE}} = \theta_u\,U(t), \qquad \theta_u \in [-\underline{\theta}_u,\, 0]$$

and similarly

$$\theta_u > \tau_u(P', t) := -\frac{\hat{S}(P', t)}{U(t)}$$

where θ_u is drawn from distribution Γ_u. Acceptance is then realized with probability

$$\Psi^{\text{UE}}(P', t) = \Pr\left(\theta_u > -\frac{\hat{S}(P', t)}{U(t)}\right) = \bar{\Gamma}_u\left(-\frac{\hat{S}(P', t)}{U(t)}\right).$$

The promise at the destination equals the second price against unemployment, $R^U(\theta_u, t) = U(t) - \xi^{UE} = U(t)(1 - \theta_u)$. Note that this leaves the total value of the worker unchanged at $\mathcal{V}^U = U(t)$.

UE and EE contact allocation and Unemployment. Unemployed workers receive firm offers at Poisson intensity $\lambda_u(t)$. UE contact market clearing requires

$$\lambda_u(t)\,u_t\,N_t^I \;=\; \lambda(1 - s)N_t^J$$

Hence $\lambda_u(t) = \frac{\lambda(1-s)N_t^J}{u_t N_t^I}$.

Total economy-wide EE contacts per unit time are

$$C(t) \;=\; \lambda\, s\, N_t^J.$$

Each contact selects an outside firm j with equal probability

$$q_j(t) \;=\; \frac{1}{N_t^J}.$$

Within the chosen firm, one worker is selected uniformly at random, so the per–worker EE offer hazard in firm j is

$$h_j(t) \;=\; \frac{C(t)}{N_t^J}\cdot\frac{1}{N_j} \;=\; \frac{\lambda\, s}{N_j}.$$

Aggregating over the N_j workers in firm j, the firm–level EE offer hazard is

$$H_j(t) \;=\; N_j\, h_j(t) \;=\; \lambda\, s,$$

which is independent of firm size. Each EE initiation picks a target firm uniformly across firms; hence the contacted incumbent's productivity is drawn from the firm productivity distribution $G(P,t)$. Economy–wide UE hires per unit time are

$$\chi(u_t)\,\lambda(1-s)\,N_t^J \int \Psi^{UE}(P,t)dG(P,t)$$

With one worker per firm separating at Poisson rate δ, steady-state unemployment rate u is then defined by

$$\chi(u) = \frac{\delta}{\lambda(1-s)\int \Psi^{UE}(P)dG(P)}.$$

Let $\eta_t := N_t^J/N_t^I$. With population growth $\dot{N}_t^I = \mu_w N_t^I$ and $\dot{N}_t^J = \mu_J N_t^J$, we have

$$\dot{u}_t = \eta_t\Big[\delta - \chi(u_t)\lambda(1-s)\int \Psi^{UE}(P,t)dG(P,t)\Big] - \mu_w\, u_t, \qquad \dot{\eta}_t = (\mu_J - \mu_w)\,\eta_t.$$

B.2.2 Worker HJB.

Let $\mathcal{W}^\star(P,N,\mathcal{R},t)$ be the worker's value under continuous renegotiation and $w(P,N,\mathcal{R},t)$ their wage. The worker faces a series of Poisson rate shocks: (i) a single firm-level EU shock at rate δ (split into own vs. others), (ii) a firm-level UE-contact rate $\lambda(1-s)\chi(u)$ (others only), (iii) a firm-level EE-contact rate λs aimed at hiring outsiders (others only), (iv) a firm-level innovation at rate β (others only), and (v) EE-offer at rate λs split into own vs.

others). The corresponding worker HJB is then

$$
\begin{aligned}
r\,\mathcal{W}^\star(P,N,\mathcal{R},t) = w(P,N,\mathcal{R},t) + \dot{\mathcal{W}}^\star(P,N,\mathcal{R},t) + \underbrace{\frac{\delta}{N}\Big(U(t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)}_{\text{own EU exit}} \\
+ \underbrace{\frac{\delta(N-1)}{N}\Big(\mathcal{W}^\star(P,N-1,\mathcal{R},t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)}_{\text{other worker EU exits }\Rightarrow N\downarrow} \\
+ \underbrace{\lambda(1-s)\chi(u)\iint \mathbf{1}\{\theta_u > \tau_u(P,t)\} \Big(\mathcal{W}^\star(\max\{P,P'\},N+1,\mathcal{R},t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dF(P',t)d\Gamma_u(\theta_u)}_{\text{other worker UE hire }\Rightarrow N\uparrow\text{; possible }P\text{-adoption}} \\
+ \underbrace{\lambda s\iint \mathbf{1}\{\theta \;> \tau(P,P',t)\}\Big(\mathcal{W}^\star(\max\{P,P'\},N+1,\mathcal{R},t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dG(P',t)d\Gamma(\theta)}_{\text{other worker EE hire }\Rightarrow N\uparrow\text{; possible }P\text{-adoption}} \\
+ \underbrace{\lambda s\,\tfrac{N-1}{N}\int\!\!\int \mathbf{1}\{\theta \;> \tau(P',P,t)\}\Big(\mathcal{W}^\star(P,N{-}1,\mathcal{R},t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dG(P',t)d\Gamma(\theta)}_{\text{other worker EE outflow }\Rightarrow N\downarrow} \\
+ \underbrace{\lambda\frac{s}{N}\iint \mathbf{1}\{\theta \;> \tau(P',P,t)\}\Big(\mathcal{V}(P,t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dG(P',t)d\Gamma(\theta)}_{\text{own EE offer accepted: second-price promise}} \\
+ \underbrace{\lambda\frac{s}{N}\iint \mathbf{1}\{\theta \;\le \tau(P',P,t)\}\Big(\mathcal{W}^\star\big(P,N,\,\mathcal{R}^+(P',P,\theta,\mathcal{R},t),\,t\big) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dG(P',t)d\Gamma(\theta)}_{\text{own EE offer rejected: promise update}} \\
+ \underbrace{\beta\int_P^\infty \Big(\mathcal{W}^\star(P',N,\mathcal{R},t) - \mathcal{W}^\star(P,N,\mathcal{R},t)\Big)\,dV(P',t)}_{\text{innovation}}
\end{aligned}
\tag{8}
$$

B.2.3 Firm value

Let $J(P,N,\boldsymbol{\mathcal{R}},t)$ denote the expected value of a firm with productivity P, size N and vector of workers' promised utilities $\boldsymbol{\mathcal{R}} = (\mathcal{R}_1,\dots,\mathcal{R}_N)$. Realized flow output is PN. Payroll is $\mathcal{W}^\Sigma(P,N,\boldsymbol{\mathcal{R}},t) := \sum_{k=1}^N w(P,N,\mathcal{R}_k,t)$.

The second-price schedule is $\mathcal{R}(P,\theta,t) := U(t) + \hat{S}(P,t)(1-\theta)$ with total value of the worker $\mathcal{V}(P,t) = U(t) + \hat{S}(P,t)$.

We define for worker k:

$$\begin{aligned}
&\boldsymbol{\mathcal{R}}^{-k} : \text{ remove } \mathcal{R}_k, \quad \boldsymbol{\mathcal{R}} \oplus \{x\} : \text{ append promise } x,\\
&\boldsymbol{\mathcal{R}}^{(k)\uparrow}(P'',\theta) := \big(\mathcal{R}_1, \dots, \mathcal{R}^{+}(P'',P,\theta,\mathcal{R},t), \dots, \mathcal{R}_N\big),
\end{aligned}$$

The corresponding firm HJB is then

$$\begin{aligned}
r\, J(P,N,\boldsymbol{\mathcal{R}},t) = {} & PN - \mathcal{W}^{\Sigma}(P,N,\boldsymbol{\mathcal{R}},t) + \dot{J}(P,N,\boldsymbol{\mathcal{R}},t)\\
& + \underbrace{\delta\Big(\frac{1}{N}\sum_{k=1}^{N} J(P,N-1,\boldsymbol{\mathcal{R}}^{-k},t) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)}_{\text{one firm-level EU rate}}\\
& + \underbrace{\begin{aligned}&\lambda\,(1-s)\chi(u_t)\iint \mathbf{1}\{\theta_u > \tau_u(P,t)\}\\ &\Big(J\big(\max\{P,P'\},\, N+1,\, \boldsymbol{\mathcal{R}} \oplus \{U(t)\}, t\big) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dF(P',t) d\Gamma_u(\theta_u)\end{aligned}}_{\text{UE inflow (promise } U\text{), adopt if } P'>P}\\
& + \underbrace{\begin{aligned}&\lambda\, s \iint \mathbf{1}\big\{\theta \; > \tau(P,P'',t)\big\}\\ &\quad\Big(J\big(\max\{P,P''\},\, N+1,\, \boldsymbol{\mathcal{R}} \oplus \{R(P'',\theta,t)\},\; t\big) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dG(P'',t) d\Gamma(\theta)\end{aligned}}_{\text{EE inflow from other firms \& possible adoption}}\\
& + \underbrace{\begin{aligned}&\frac{\lambda s}{N}\sum_{k=1}^{N}\int\int \mathbf{1}\big\{\theta \; > \tau(P'',P,t)\big\}\\ &\Big(J(P,N-1,\boldsymbol{\mathcal{R}}^{-k},t) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dG(P'',t) d\Gamma(\theta)\end{aligned}}_{\text{own workers' EE departures (accepted)}}\\
& + \underbrace{\begin{aligned}&\frac{\lambda s}{N}\sum_{k=1}^{N}\int\int \mathbf{1}\big\{\theta \; \le \tau(P'',P,t)\big\}\\ &\Big(J\big(P,N,\boldsymbol{\mathcal{R}}^{(k)\uparrow}(P'',\theta),t\big) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dG(P'',t) d\Gamma(\theta)\end{aligned}}_{\text{own workers' EE rejections (upward promise)}}\\
& + \underbrace{\beta \int_{P}^{\infty} \Big(J(P',N,\boldsymbol{\mathcal{R}},t) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dV(P',t)}_{\text{innovation (adopt to } P' \text{ if higher)}}\,.
\end{aligned} \tag{9}$$

B.2.4 Coalition HJB Ω (row-by-row mapping)

Define the coalition value

$$\Omega(P,N,\boldsymbol{\mathcal{R}},t) := J(P,N,\boldsymbol{\mathcal{R}},t) \; + \; \sum_{k=1}^{N} \mathcal{W}^{\star}(P,N,\mathcal{R}_k,t),$$

We now examine all blocks of the coalition value one by one.

Block A: Deterministic flows.

$$\begin{aligned}
\text{Firm:} \quad & PN - \mathcal{W}^{\Sigma}(P,N,\boldsymbol{\mathcal{R}},t) + \dot{J}(P,N,\boldsymbol{\mathcal{R}},t), \\
\text{Workers:} \quad & \sum_{k=1}^{N} w(P,N,\mathcal{R}_k,t) \; + \; \sum_{k=1}^{N} \dot{\mathcal{W}}^{\star}(P,N,\mathcal{R}_k,t) \\
\Delta\Omega_{\text{flow}} &= PN + \dot{\Omega}(P,N,\boldsymbol{\mathcal{R}},t)
\end{aligned}$$

Block B: EU separations (one firm-level clock δ).

$$\text{Firm:} \quad \delta\Big(\tfrac{1}{N}\sum_{k=1}^{N} J(P,N{-}1,\boldsymbol{\mathcal{R}}^{-k},t) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big),$$

$$\text{Workers:} \quad \sum_{k=1}^{N}\Big[\tfrac{\delta(N-1)}{N}\big(\mathcal{W}^{\star}(P,N{-}1,\mathcal{R}_k,t) - \mathcal{W}^{\star}(P,N,\mathcal{R}_k,t)\big) + \tfrac{\delta}{N}\big(U(t) - \mathcal{W}^{\star}(P,N,\mathcal{R}_k,t)\big)\Big]$$

$$\Delta\Omega_{\text{EU}} = \tfrac{\delta}{N}\sum_{\ell=1}^{N}\big(\Omega(P,N{-}1,\boldsymbol{\mathcal{R}}^{-\ell},t) - \Omega(P,N,\boldsymbol{\mathcal{R}},t)\big) \; + \; \delta\, U(t)$$

Block C: UE contacts (firm-level rate $\lambda(1-s)\chi(u)$). Let $P^{+} = \max\{P,P'\}$ and $N^{+} = N+1$ and acceptance indicator $\mathbf{1}\{\cdot\} := \mathbf{1}\{\theta_u > \tau_u(P,t)\}$

$$\begin{aligned}
\text{Firm:} \quad & \lambda(1-s)\chi(u)\iint 1\{\cdot\}\Big(J(P^{+},N^{+},\boldsymbol{\mathcal{R}}\oplus\{U(t)\},t) - J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\, dF(P',t)d\Gamma_u(\theta_u), \\
\text{Incumbent workers:} \quad & \lambda(1-s)\chi(u)\iint\sum_{k=1}^{N} 1\{\cdot\}\Big(\mathcal{W}^{\star}(P^{+},N^{+},\mathcal{R}_k,t) - \mathcal{W}^{\star}(P,N,\mathcal{R}_k,t)\Big)\, dF(P',t)d\Gamma_u(\theta_u), \\
\text{Joiner:} \quad & \lambda(1-s)\chi(u)\iint 1\{\cdot\}\Big(\mathcal{W}^{\star}(P^{+},N^{+},U(t),t) - U(t)\Big)\, dF(P',t)d\Gamma_u(\theta_u)
\end{aligned}$$

$$\Delta\Omega_{\text{UE}} = \lambda(1{-}s)\chi(u)\iint 1\{\cdot\}\Big(\Omega(P^{+},N^{+},\boldsymbol{\mathcal{R}}\oplus\{U(t)\},t) {-} \Omega(P,N,\boldsymbol{\mathcal{R}},t) {-} U(t)\Big)\, dF(P',t)d\Gamma_u(\theta_u)$$

Block D: EE inflows to this firm (firm-level rate λs). Let $P^+ = \max\{P, P'\}$, $N^+ = N + 1$, acceptance indicator $\mathbf{1}\{\cdot\} := \mathbf{1}\{\theta \ > \tau(P, P', t)\}$, and $\mathcal{R}^{\text{new}} = \mathcal{R}(P', \theta, t)$.

$$\text{Firm:} \quad \lambda s \iint \mathbf{1}\{\cdot\} \Big(J(P^+, N^+, \boldsymbol{\mathcal{R}} \oplus \{R^{\text{new}}\}, t) - J(P, N, \boldsymbol{\mathcal{R}}, t) \Big) \, dG(P', t) \, d\Gamma(\theta),$$

$$\text{Incumbent workers:} \quad \lambda s \iint \mathbf{1}\{\cdot\} \sum_{k=1}^{N} \Big(\mathcal{W}^\star(P^+, N^+, \mathcal{R}_k, t) - \mathcal{W}^\star(P, N, \mathcal{R}_k, t) \Big) \, dG(P', t) \, d\Gamma(\theta),$$

$$\text{Joiner:} \quad \lambda s \iint \mathbf{1}\{\cdot\} \Big(\mathcal{W}^\star(P^+, N^+, R^{\text{new}}, t) - \mathcal{V}(P', t) \Big) \, dG(P', t) \, d\Gamma(\theta)$$

$$\Delta\Omega_{\text{EE,in}} = \lambda s \iint \mathbf{1}\{\cdot\} \Big(\Omega(P^+, N^+, \boldsymbol{\mathcal{R}} \oplus \{R^{\text{new}}\}, t) - \Omega(P, N, \boldsymbol{\mathcal{R}}, t) \Big) \, dG(P', t) \, d\Gamma(\theta) \ - \ \mathcal{V}_{\text{dest}} \ ,$$

where

$$\mathcal{V}_{\text{dest}} := \lambda s \iint \mathbf{1}\{\cdot\} \, \mathcal{V}(P', t) \, dG(P', t) \, d\Gamma(\theta).$$

Block E: EE outflows from this firm (per–worker rate $\lambda s/N$). Acceptance to destination P'' iff $\mathbf{1}\{\cdot\} := \mathbf{1}\{\theta \ > \tau(P'', P, t)\}$.

$$\text{Firm:} \quad \frac{\lambda s}{N} \sum_{k=1}^{N} \iint \mathbf{1}\{\cdot\} \Big(J(P, N{-}1, \boldsymbol{\mathcal{R}}^{-k}, t) - J(P, N, \boldsymbol{\mathcal{R}}, t) \Big) \, dG(P'', t) \, d\Gamma(\theta),$$

$$\text{Co-workers:} \quad \frac{\lambda s}{N} \sum_{k=1}^{N} \iint \mathbf{1}\{\cdot\} \sum_{i \neq k} \Big(\mathcal{W}^\star(P, N{-}1, \mathcal{R}_i, t) - \mathcal{W}^\star(P, N, \mathcal{R}_i, t) \Big) \, dG(P'', t) \, d\Gamma(\theta),$$

$$\text{Leaver:} \quad \frac{\lambda s}{N} \sum_{k=1}^{N} \iint \mathbf{1}\{\cdot\} \Big(\mathcal{V}(P, t) - \mathcal{W}^\star(P, N, \mathcal{R}_k, t) \Big) \, dG(P'', t) \, d\Gamma(\theta)$$

$$\Delta\Omega_{\text{EE,out}} = \frac{\lambda s}{N} \sum_{k=1}^{N} \iint \mathbf{1}\{\cdot\} \Big(\Omega(P, N{-}1, \boldsymbol{\mathcal{R}}^{-k}, t) - \Omega(P, N, \boldsymbol{\mathcal{R}}, t) \Big) \, dG(P'', t) \, d\Gamma(\theta) \ + \ \mathcal{V}_{\text{orig}} \ ,$$

where

$$\mathcal{V}_{\text{orig}} := \frac{\lambda s}{N} \sum_{k=1}^{N} \iint \mathbf{1}\{\cdot\} \, \mathcal{V}(P, t) \, dG(P'', t) \, d\Gamma(\theta).$$

Block F: Innovation (firm-level rate β).

$$\text{Firm: } \beta\int_P^\infty \Big(J(P',N,\boldsymbol{\mathcal{R}},t)-J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dV(P',t),$$

$$\text{Workers: } \beta\int_P^\infty \sum_{k=1}^{N}\Big(\mathcal{W}^\star(P',N,\mathcal{R}_k,t)-\mathcal{W}^\star(P,N,\mathcal{R}_k,t)\Big)\,dV(P',t)$$

$$\Delta\Omega_{\text{innov}}=\beta\int_P^\infty \Big(\Omega(P',N,\boldsymbol{\mathcal{R}},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dV(P',t)\ .$$

Block G: Rejected EE offers (own workers; per–worker rate $\lambda s/N$). Rejection for destination P'' iff $\mathbf{1}\{\cdot\} := \mathbf{1}\{\theta\ \le \tau(P'',P,t)\}$; promise update $\mathcal{R}_k^+ = \max\big\{\mathcal{R},\, U(t)+\hat{S}(P'',t)+\theta\hat{S}(P,t)\big\}$.

$$\text{Firm: } \frac{\lambda s}{N}\sum_{k=1}^{N}\iint \mathbf{1}\{\cdot\}\Big(J(P,N,\boldsymbol{\mathcal{R}}^{(k)\uparrow}(P'',\theta),t)-J(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dG(P'',t)\,d\Gamma(\theta),$$

$$\text{Workers: } \frac{\lambda s}{N}\sum_{k=1}^{N}\iint \mathbf{1}\{\cdot\}\Big(\mathcal{W}^\star(P,N,\mathcal{R}_k^+,t)-\mathcal{W}^\star(P,N,\mathcal{R}_k,t)\Big)\,dG(P'',t)\,d\Gamma(\theta)$$

$$\Delta\Omega_{\text{Reneg}}=\frac{\lambda s}{N}\sum_{k=1}^{N}\iint \mathbf{1}\{\cdot\}\Big(\Omega(P,N,\boldsymbol{\mathcal{R}}^{(k)\uparrow}(P'',\theta),t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dG(P'',t)\,d\Gamma(\theta)$$

Full coalition HJB

Collecting Blocks A–G:

$$
\begin{aligned}
r\,\Omega(P,N,\boldsymbol{\mathcal{R}},t) = \; & \underbrace{PN}_{\text{realized flow}} \; + \dot{\Omega}(P,N,\boldsymbol{\mathcal{R}},t) \\
& \underbrace{+\frac{\delta}{N}\sum_{\ell=1}^{N}\Big(\Omega(P,N{-}1,\boldsymbol{\mathcal{R}}^{-\ell},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\;+\;\delta\,U(t)}_{\text{EU (single firm clock)}} \\
& \underbrace{+\begin{array}{l}\lambda(1-s)\chi(u)\iint 1\{\theta_u>\tau_u(P,t)\} \\ \Big(\Omega(\max\{P,P'\},N{+}1,\boldsymbol{\mathcal{R}}\oplus\{U(t)\},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)-U(t)\Big)\,dF(P',t)d\Gamma_u(\theta_u)\end{array}}_{\text{UE inflow (hire \& possible adoption)}} \\
& \underbrace{+\begin{array}{l}\lambda s\iint \mathbf{1}\big\{\theta\;>\tau(P,P',t)\big\} \\ \Big(\Omega(\max\{P,P'\},N{+}1,\boldsymbol{\mathcal{R}}\oplus\{R(P',\theta,t)\},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dG(P',t)\,d\Gamma(\theta)\end{array}}_{\text{EE inflow (destination) value change}} \\
& \underbrace{+\frac{\lambda s}{N}\sum_{k=1}^{N}\iint \mathbf{1}\big\{\theta\;>\tau(P'',P,t)\big\}\Big(\Omega(P,N{-}1,\boldsymbol{\mathcal{R}}^{-k},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dG(P'',t)\,d\Gamma(\theta)}_{\text{EE outflow (origin) value change}} \\
& \underbrace{+\beta\int_{P}^{\infty}\Big(\Omega(P',N,\boldsymbol{\mathcal{R}},t)-\Omega(P,N,\boldsymbol{\mathcal{R}},t)\Big)\,dV(P',t)}_{\text{innovation (adopt if } P'>P)} \\
& \underbrace{-\lambda s\iint \mathbf{1}\big\{\theta\;>\tau(P,P',t)\big\}\;\mathcal{V}(P',t)\,dG(P',t)\,d\Gamma(\theta)}_{\text{EE inflow transfer: subtract joiner's origin}} \\
& \underbrace{+\frac{\lambda s}{N}\sum_{k=1}^{N}\iint \mathbf{1}\big\{\theta\;>\tau(P'',P,t)\big\}\;\mathcal{V}(P,t)\,dG(P'',t)\,d\Gamma(\theta)}_{\text{EE outflow transfer: add leaver's second price total utility at destination}}\;. \qquad (10)
\end{aligned}
$$

We can simplify this expression using three insights. First, note that the vector of promised utilities does not affect the joint value. Second, the difference in value due to an EE outflow $\Big(\Omega(P,N{-}1,t)-\Omega(P,N,t)\Big)$ is completely compensated, due to sequential auctions with second price value, by the worker's total value at the next job $\mathcal{V}(P,t)$. Third, using the fact that the distribution of mobility shocks is independent of the distribution of productivity, we can factor out the probability of a successful match. Hence we can write

$$
\begin{aligned}
r\,\Omega(P,N,t) &= PN + \dot{\Omega}(P,N,t) \\
&\quad + \underbrace{\frac{\delta}{N}\sum_{\ell=1}^{N}\Big(\Omega(P,N{-}1,t) - \Omega(P,N,t) + U(t)\Big)}_{\text{EU (single firm clock)}} \\
&\quad + \underbrace{\lambda(1-s)\chi(u)\int \Psi^{\mathrm{UE}}(P,t)\Big(\Omega(\max\{P,P'\},N{+}1,t) - \Omega(P,N,t) - U(t)\Big)\,dF(P',t)}_{\text{UE inflow (hire \& possible adoption)}} \\
&\quad + \underbrace{\lambda s\int \Psi(P,P',t)\Big(\Omega(\max\{P,P'\},N{+}1,t) - \Omega(P,N,t) - \mathcal{V}(P',t)\Big)\,dG(P',t)}_{\text{EE inflow (destination) value change}} \\
&\quad + \underbrace{\beta\int_{P}^{\infty}\Big(\Omega(P',N,t) - \Omega(P,N,t)\Big)\,dV(P',t)}_{\text{innovation (adopt if } P'>P)}
\end{aligned}
$$

It is useful to express the coalition value in terms of worker-firm surplus. Define $S(P,N,t) := \Omega(P,N,t) - N\,U(t)$ and use $rU(t) = b(t) + \dot{U}(t)$. Write the discrete size difference and the productivity-jump difference as

$$
\Delta_N S(P,N,t) := S(P,N,t) - S(P,N-1,t), \qquad S_{P',P}(N,t) := S(P',N,t) - S(P,N,t).
$$

Starting from the coalition HJB, subtracting $N\,U(t)$ and collecting terms block by block, the surplus HJB is

$$
\begin{array}{rcl}
rS(P,N,t) & = & N\,(P - b^{u}(t)) + \dot{S}(P,N,t) \\
\text{[EU Mobility]} & + & \delta\,(\,-\Delta_N S(P,N,t)\,) \\
\text{[UE \& EE Mobility]} & + & \lambda(1-s)\chi(u)\Psi^{\mathrm{UE}}(P,t)\left(\displaystyle\int_{P}^{\infty} S_{P',P}(N,t)\,dF(P',t) + \Delta_N S(P,N,t)\right) \\
 & + & \lambda s\displaystyle\int_{P}^{\infty}\Psi(P,P',t)\;S_{P',P}(N,t)\,dG(P',t) \\
 & + & \lambda s\displaystyle\int_{\underline{P}}^{\infty}\Psi(P,P',t)(\hat{S}(P,t) - \hat{S}(P',t))\,dG(P',t) \\
\text{[Innovation]} & + & \beta\displaystyle\int_{P}^{\infty} S_{P',P}(N,t)\,dV(P',t)
\end{array}
$$

where the fifth line uses the affine surplus representation.

B.3 Affine Surplus Representation

Start from the surplus HJB with EE contacts initiated by the destination firm (rate λs) with acceptance rate of a job offer at firm P with incumbent firm quality P' ,$\Psi(P, P', t)$:

$$
\begin{aligned}
rS(P,N,t) \quad &= \quad N\left(P - b^u(t)\right) + \dot{S}(P,N,t) \\
\text{[EU Mobility]} \quad &+ \quad \delta\left(-\Delta_N S(P,N,t)\right) \\
\text{[UE \& EE Mobility]} \quad &+ \quad \lambda(1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\left(\int_P^\infty S_{P',P}(N,t)\,dF(P',t) + \Delta_N S(P,N,t)\right) \\
&+ \quad \lambda s\int_P^\infty \Psi(P,P',t)\; S_{P',P}(N,t)\,dG(P',t) \\
&+ \quad \lambda s\int_{\underline{P}}^\infty \Psi(P,P',t)(\hat{S}(P,t) - \hat{S}(P',t))\,dG(P',t) \\
\text{[Innovation]} \quad &+ \quad \beta\int_P^\infty S_{P',P}(N,t)\,dV(P',t)
\end{aligned}
$$

Under the guess $S(P,N,t) = N\hat{S}(P,t) + S^0(P,t)$,

$$
\Delta_N S(P,N,t) = \hat{S}(P,t), \qquad \dot{S}(P,N,t) = N\,\dot{\hat{S}}(P,t) + \dot{S}^0(P,t),
$$

and for any $P' \geq P$,

$$
S_{P',P}(N,t) = N\,\hat{S}_{P',P}(t) + S^0_{P',P}(t), \quad \hat{S}_{P',P}(t) := \hat{S}(P',t) - \hat{S}(P,t), \quad S^0_{P',P}(t) := S^0(P',t) - S^0(P,t).
$$

Plugging in the affine guess and collecting coefficients gives:
(i) Slope equation.

$$
\begin{aligned}
r\,\hat{S}(P,t) = \big(P - b^u(t)\big) + \dot{\hat{S}}(P,t) + \lambda(1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\int_P^\infty \hat{S}_{P',P}(t)\,dF(P',t) \\
+\lambda s\int_P^\infty \Psi(P,P',t)\,\hat{S}_{P',P}(t)\,dG(P',t) + \beta\int_P^\infty \hat{S}_{P',P}(t)\,dV(P',t).
\end{aligned}
$$

(ii) Intercept equation.

$$
\begin{aligned}
r\,S^0(P,t) = \dot{S}^0(P,t) - \delta\,\hat{S}(P,t) + \lambda(1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\,\hat{S}(P,t) \\
+\lambda\left((1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\int_P^\infty S^0_{P',P}(t)\,dF(P',t) + s\int_P^\infty \Psi(P,P',t)\,S^0_{P',P}(t)\,dG(P',t)\right) \\
+\lambda s\int_{\underline{P}}^\infty \Psi(P,P',t)\left[\hat{S}(P,t) - \hat{S}(P',t)\right]dG(P',t) + \beta\int_P^\infty S^0_{P',P}(t)\,dV(P',t)
\end{aligned}
$$

Thus the affine representation in firm size holds. Note that the slope equation is a non-linear equation due to $\Psi(P, P', t)$ whereas the intercept equation is linear conditional on the slope equation.

B.4 Characteristics of marginal surplus

Starting from the equilibrium equation for the slope term in the affine representation of surplus (see Appendix B.3),

$$\begin{aligned} r\,\hat{S}(P,t) \; &= \big(P - b^u(t)\big) + \dot{\hat{S}}(P,t) + \lambda(1-s)\chi(u_t)\Psi^{\mathrm{UE}}(P,t)\textstyle\int_P^\infty \hat{S}_{P',P}(t)\,dF(P',t) \\ &\quad + \lambda s \textstyle\int_P^\infty \Psi(P,P',t)\,\hat{S}_{P',P}(t)\,dG(P',t) + \beta \textstyle\int_P^\infty \hat{S}_{P',P}(t)\,dV(P',t), \end{aligned} \tag{11}$$

we study existence, uniqueness, and monotonicity of the marginal surplus $\hat{S}(P,t)$.

Throughout this subsection we fix a calendar date t and suppress the explicit time argument when this does not create confusion. In particular we write $b^u = b^u(t)$, $u = u_t$, and $\hat{S}(P) = \hat{S}(P,t)$. The arguments below apply pointwise in t. For notational clarity we also recall that, as in Appendix B.3, the option-value terms can be written as

$$\hat{S}_{P',P}(t) \;=\; \hat{S}(P',t) - \hat{S}(P,t).$$

We exploit a key structural property of (11): all integrals run over $P' \geq P$. Hence, for a given P, only the continuation values at higher productivity levels matter. This triangular structure allows us to construct the solution from the top of the productivity ladder without requiring assumptions on discounting.

Assumption 1 (Productivity ladder and arrivals) *(i) Productivity lies in a compact interval $P \in [\underline{P}, \bar{P}]$. (ii) For each fixed t, the measures $F(\cdot,t)$, $G(\cdot,t)$, and $V(\cdot,t)$ put no mass above $\bar{P}$, and have bounded total mass on $[\underline{P}, \bar{P}]$. (iii) The functions $\chi(u_t)$, $\Psi^{\mathrm{UE}}(P,t)$, $\Psi(P,P',t)$, and the arrival intensities λ, β are bounded, $r > 0$.*

Existence and uniqueness via traingular structure

To highlight the triangular structure in P, it is convenient to rewrite (11) in a stationary cross-section, dropping $\dot{\hat{S}}$ and the explicit time argument. Using $\hat{S}_{P',P} = \hat{S}(P') - \hat{S}(P)$, we obtain

$$r\,\hat{S}(P) = \big(P - b^u\big) + \lambda(1-s)\chi(u)\Psi^{\mathrm{UE}}(P)\int_P^{\bar{P}} [\hat{S}(P') - \hat{S}(P)]\,dF(P')$$
$$+ \lambda s \int_P^{\bar{P}} \Psi(P,P')\,[\hat{S}(P') - \hat{S}(P)]\,dG(P') + \beta \int_P^{\bar{P}} [\hat{S}(P') - \hat{S}(P)]\,dV(P'). \qquad (12)$$

For any finite measure μ and bounded function ϕ,

$$\int_P^{\bar{P}} [\phi(P') - \phi(P)]\,d\mu(P') = \int_P^{\bar{P}} \phi(P')\,d\mu(P') - \phi(P)\,\mu([P,\bar{P}]).$$

Applying this identity term-by-term in (12) yields

$$\big[r + A(P)\big]\hat{S}(P) = \big(P - b^u\big) + B(P), \qquad (13)$$

where

$$A(P) := \lambda(1-s)\chi(u)\Psi^{\mathrm{UE}}(P)\,F([P,\bar{P}]) + \lambda s \int_P^{\bar{P}} \Psi(P,P')\,dG(P') + \beta\,V([P,\bar{P}]),$$

$$B(P) := \lambda(1-s)\chi(u)\Psi^{\mathrm{UE}}(P)\int_P^{\bar{P}} \hat{S}(P')\,dF(P') + \lambda s \int_P^{\bar{P}} \Psi(P,P')\,\hat{S}(P')\,dG(P')$$
$$+ \beta \int_P^{\bar{P}} \hat{S}(P')\,dV(P').$$

By construction $A(P) \geq 0$ for all P, and, by Assumption 1(i), $A(\bar{P}) = 0$ and $B(\bar{P}) = 0$.

Result 2 (Existence and uniqueness of marginal surplus) *Suppose Assumption 1 holds. For every fixed t, the equilibrium equation (12) admits a unique bounded solution $P \mapsto \hat{S}(P,t)$ on $[\underline{P},\bar{P}]$. Moreover, this solution can be constructed recursively from the top of the productivity ladder.*

Proof Sketch: At the top productivity level $P = \bar{P}$, Assumption 1 implies $A(\bar{P}) = 0$ and $B(\bar{P}) = 0$, so

$$r\,\hat{S}(\bar{P}) = \bar{P} - b^u,$$

and hence

$$\hat{S}(\bar{P}) = \frac{\bar{P} - b^u}{r}.$$

Thus the marginal surplus of the most productive firm is uniquely determined and depends only on its flow payoff.

Now fix $P < \bar{P}$. By Assumption 1, the measures F, G, V put no mass above $\bar{P}$, and the integrands involve only $\hat{S}(P')$ with $P' \geq P$. In particular, once $\hat{S}(P')$ is known for all $P' > P$, $B(P)$ is completely determined. Hence (13) becomes a single linear equation in the scalar unknown $\hat{S}(P)$:

$$\big[r + A(P)\big]\hat{S}(P) = \big(P - b^u\big) + B(P),$$

with $r + A(P) > 0$. Therefore $\hat{S}(P)$ is uniquely determined by

$$\hat{S}(P) = \frac{P - b^u + B(P)}{r + A(P)}.$$

Starting from $P = \bar{P}$, we can apply this argument successively to lower productivity levels. In a discrete approximation with grid $\underline{P} = P_1 < \cdots < P_N = \bar{P}$, the system is lower triangular in $\{\hat{S}(P_i)\}_{i=1}^N$ and hence admits a unique solution obtained by backward substitution.

Monotonicity in productivity

Result 3 (Monotonicity of marginal surplus in productivity) *Suppose Assumption 1 holds. For every fixed t, the unique solution $P \mapsto \hat{S}(P,t)$ to (12) is strictly increasing in P: if $P' > P$, then $\hat{S}(P',t) > \hat{S}(P,t)$.*

We show that the unique solution constructed above is strictly increasing in productivity. Intuitively, a more productive firm has a higher flow payoff $P - b^u$ and, because productivity moves only upward along the ladder, it starts from a strictly better position in any future history. The option value of future moves cannot overturn this ordering: for every state, the option value is bounded between zero (never moving up) and the surplus of a firm that is already at the top rung $\bar{P}$.[14]

For the formal argument we again work with the discrete grid $\underline{P} = P_1 < \cdots < P_N = \bar{P}$ and write $S_i \equiv \hat{S}(P_i, t)$. The discrete counterpart of (12) reads

$$r\,S_i = (P_i - b^u) + \sum_{j>i} a_{ij}\,\big(S_j - S_i\big), \qquad i = 1, \ldots, N, \tag{14}$$

with $a_{ij} \geq 0$ collecting arrival rates and transition probabilities from P_i to higher productivities

[14] Formally, the Bellman equation $r\hat{S}(P) = (P - b^u) + \mathrm{OV}(P)$ with a nonnegative option value term $\mathrm{OV}(P)$ implies $(P - b^u)/r \leq \hat{S}(P) \leq (\bar{P} - b^u)/r$ for all $P \in [\underline{P}, \bar{P}]$. In particular, the option value can never lift a low-productivity firm above the surplus of the most productive firm; it only compresses differences in P.

P_j. For the top state $i = N$ there are no upward moves, so $a_{Nj} = 0$ for all j and

$$r\, S_N = P_N - b^u \quad \Rightarrow \quad S_N = \frac{P_N - b^u}{r}.$$

The key observation is that (14) is exactly the Bellman equation of a continuous-time Markov chain on $\{1, \ldots, N\}$, with state i corresponding to productivity P_i, instantaneous payoff $P_i - b^u$, and only upward transitions ($j > i$) allowed. Standard dynamic programming arguments then imply the representation

$$S_i = E_i\left[\int_0^\infty e^{-r\tau}\big(P_{J_\tau} - b^u\big)\, d\tau\right], \tag{15}$$

where $(J_\tau)_{\tau \geq 0}$ is the Markov chain on $\{1, \ldots, N\}$ induced by the coefficients a_{ij} and $E_i[\cdot]$ denotes expectation conditional on $J_0 = i$.
Because transitions are only upward, we can couple any two chains starting from different initial productivities in an order-preserving way. Fix $i < k$ and construct the processes $\big(J_\tau^{(i)}, J_\tau^{(k)}\big)_{\tau \geq 0}$ on a common probability space so that

$$J_0^{(i)} = i, \qquad J_0^{(k)} = k,$$

and both chains share the same jump times and destination draws. Since all jumps are to strictly higher indices, this coupling satisfies

$$J_\tau^{(i)} \leq J_\tau^{(k)} \quad \text{for all } \tau \geq 0 \quad \text{almost surely.}$$

As the payoff $P_j - b^u$ is strictly increasing in j, we obtain

$$P_{J_\tau^{(i)}} - b^u \;\leq\; P_{J_\tau^{(k)}} - b^u \quad \text{for all } \tau \geq 0,$$

with strict inequality on a set of τ of positive measure whenever $P_k > P_i$. Integrating over time and taking expectations in (15) yields

$$S_k - S_i = E\left[\int_0^\infty e^{-r\tau}\big(P_{J_\tau^{(k)}} - P_{J_\tau^{(i)}}\big)\, d\tau\right] > 0 \quad \text{for all } k > i,$$

so the discrete approximation to $\hat{S}(\cdot, t)$ is strictly increasing in productivity.

Dispersion and marginal surplus

Fix t and write $\hat{S}(P;\sigma)$. On a grid $\underline{P} = P_1 < \cdots < P_J = \bar{P}$, the stationary equation (12) becomes

$$r\,S_j = (P_j - b^u) + \sum_{k>j} a^{\mathrm{UE}}_{jk}(\sigma)\,(S_k - S_j) + \sum_{k>j} a^{\mathrm{EE}}_{jk}(\sigma)\,(S_k - S_j) + \sum_{k>j} a^{\mathrm{I}}_{jk}\,(S_k - S_j), \quad (16)$$

where $S_j = \hat{S}(P_j;\sigma)$, $a^{\mathrm{EE}}_{jk}(\sigma) = \lambda s\,\Psi_\sigma(P_j, P_k)\,G_{jk}$, $a^{\mathrm{UE}}_{jk}(\sigma) = \lambda(1-s)\chi(u)\,\Psi^{\mathrm{UE}}_\sigma(P_j)\,F_{jk}$, $a^{\mathrm{I}}_{jk} = \beta V_{jk}$, and $k > j$ means $P_k > P_j$. Assume: (i) σ enters only via the scale families in $\Psi_\sigma, \Psi^{\mathrm{UE}}_\sigma$; (ii) U is P-independent; (iii) $G_{jk}, F_{jk}, V_{jk} \geq 0$ and $\hat{S}$ is strictly increasing in P.

Result 4 (Dispersion lowers $\hat{S}$ and more so at low P) *Let $h_j := \partial_\sigma S_j$ and $B_j := \partial_\sigma \ln S_j = h_j/S_j$. Then for all j: (a) $h_j \leq 0$; (b) h_j is (weakly) increasing in j; hence B_j is (weakly) increasing in j and, for any $i < j$,*

$$\frac{\partial}{\partial\sigma}\tau_{i|j}(\sigma) = \frac{S_i}{S_j}\big(B_j - B_i\big) \;\geq\; 0, \qquad \tau_{i|j} := 1 - \frac{S_i}{S_j}.$$

Proof sketch: Differentiate (16) in σ. Using $\overline{\Gamma}'(z) = -\gamma(z) \leq 0$ and, for $k > j$, $\tau_{j|k} = 1 - S_j/S_k \leq 0$, we get

$$\partial_\sigma \Psi_\sigma(P_j, P_k) = \tfrac{\tau_{j|k}}{\sigma^2}\gamma(\tfrac{\tau_{j|k}}{\sigma}) \leq 0, \quad \partial_{S_j}\Psi_\sigma \geq 0, \quad \partial_{S_k}\Psi_\sigma \leq 0,$$

and the same signs for $\Psi^{\mathrm{UE}}_\sigma$ (its argument is $-S_j/U \leq 0$). Collecting terms yields a *lower-triangular* system

$$A_j h_j - \sum_{k>j} K_{jk} h_k = d_j,$$

with $A_j > 0$, $K_{jk} \geq 0$, and $d_j \leq 0$ (pure σ effects), where d_j is (weakly) increasing in j because the sums run over $k \geq j$ with nonpositive integrands. Backward substitution from the top bin gives $h_J = d_J/A_J \leq 0$ and inductively $h_j = \frac{1}{A_j}(d_j + \sum_{k>j} K_{jk} h_k) \leq 0$ for all j, proving (a). The same recursion plus d_j increasing in j implies h_j is (weakly) increasing in j, proving (b). Dividing by S_j (strictly increasing in j) yields B_j increasing in j and the formula for $\partial_\sigma \tau_{i|j} \geq 0$.

Result 5 (Sign of the indirect term for downward pairs) *Fix t and let $\hat{S}(P;\sigma)$ be the* *unique increasing solution to* (12)*. For any $P' < P$, write*

$$\tau(P', P; \sigma) := 1 - \frac{\hat{S}(P'; \sigma)}{\hat{S}(P; \sigma)}, \qquad B(P; \sigma) := \frac{\partial}{\partial \sigma} \ln \hat{S}(P; \sigma).$$

Then

$$\frac{\partial \tau(P', P; \sigma)}{\partial \sigma} = \frac{\hat{S}(P'; \sigma)}{\hat{S}(P; \sigma)} \Big(B(P; \sigma) - B(P'; \sigma) \Big) \;\geq\; 0,$$

with strict inequality whenever $B(\cdot; \sigma)$ is strictly increasing in P. Consequently, in

$$\frac{d\Psi_\sigma}{d\sigma} = \frac{\gamma(\tau/\sigma)}{\sigma^2} \Big(\tau(P', P; \sigma) - \sigma \, \frac{\partial \tau(P', P; \sigma)}{\partial \sigma} \Big),$$

the last term $-\sigma \, \partial_\sigma \tau$ *is* nonpositive *for all downward pairs $P' < P$ (strictly negative under the same condition).*

Proof Sketch: Differentiate $\tau = 1 - \hat{S}(P')/\hat{S}(P)$ in σ:

$$\partial_\sigma \tau = -\frac{\hat{S}_\sigma(P')\hat{S}(P) - \hat{S}(P')\hat{S}_\sigma(P)}{\hat{S}(P)^2} = \frac{\hat{S}(P')}{\hat{S}(P)} \Big(B(P) - B(P') \Big).$$

By result 4, $B(\cdot; \sigma)$ is weakly increasing in P, hence $B(P) - B(P') \geq 0$ for $P' < P$, with strict inequality when upgrading margins are active. The sign claim for $-\sigma \, \partial_\sigma \tau$ follows immediately.

B.5 Growth Rate of Productivity

Preliminaries. Let $\bar{P}_t := \int_0^\infty P\, g(P,t)\, dP$ denote the cross–sectional mean of firm productivity. We write $(x)^+ = \max\{x, 0\}$. For EE mobility we adopt the knowledge–diffusion rule: if a worker moves from origin P to destination P', then the destination upgrades to $\max\{P', P\}$. Expectations are taken under the contemporaneous distributions. Multiply (2) by P and integrate over P:

$$
\begin{aligned}
\frac{d}{dt}\bar{P}_t &= \int_0^\infty P\,\frac{\partial g(P,t)}{\partial t}\, dP \\
&= \underbrace{\int P\big[\beta v(P,t)G(P,t) - \beta g(P,t)\,\bar{V}(P,t)\big]\, dP}_{\text{Innovation}} + \underbrace{\int P\big[\mu^J b(P,t) - \mu^J g(P,t)\big]\, dP}_{\text{Entry/exit}} \\
&\quad + \underbrace{\int \lambda(1-s)\chi(u_t)P\Big[f(P,t)\int_0^P \Psi^{\mathrm{UE}}(P',\emptyset,t)\, dG(P',t) - \Psi^{\mathrm{UE}}(P,\emptyset,t)\, g(P,t)\,\bar{F}(P,t)\Big]\, dP}_{\text{UE hires}} \\
&\quad + \underbrace{\int P\,\lambda s\, g(P,t)\bigg[\int_0^P \Psi(P',P,t)\, dG(P',t) - \int_P^\infty \Psi(P,P',t)\, dG(P',t)\bigg] dP}_{\text{EE knowledge diffusion}}.
\end{aligned}
$$

We now analyze the different blocks in turn.

Innovation. Write $G(P) = \int_{x\le P} dG(x)$ and $\bar{V}(P) = \int_{Q\ge P} dV(Q)$, then change of integration:

$$
\begin{aligned}
\int P\,\beta v(P)G(P)\, dP &= \beta \iint P\,\mathbf{1}\{x \le P\}\, dG(x)\, dV(P) = \beta \iint Q\,\mathbf{1}\{x \le Q\}\, dG(x)\, dV(Q), \\
\int P\,\beta g(P)\bar{V}(P)\, dP &= \beta \iint P\,\mathbf{1}\{Q \ge P\}\, dV(Q)\, dG(P) = \beta \iint x\,\mathbf{1}\{Q \ge x\}\, dV(Q)\, dG(x).
\end{aligned}
$$

Subtracting and using $Q\,\mathbf{1}\{x \le Q\} - x\,\mathbf{1}\{Q \ge x\} = (Q-x)^+$,

$$
\int P\big[\beta v(P)G(P) - \beta g(P)\bar{V}(P)\big]\, dP = \beta \iint (Q-x)^+\, dG(x)\, dV(Q) = \beta\,\mathbb{E}\big[(P'-P)^+\big].
$$

Entry/exit. Directly,

$$
\int P\big[\mu^J b(P) - \mu^J g(P)\big]\, dP = \mu^J\left(\int P\, b(P)\, dP - \int P\, g(P)\, dP\right) = \mu^J\big(\mathbb{E}_\Lambda[P] - \bar{P}_t\big).
$$

UE hires. Using $\bar{F}(P) = \int_{x \geq P} dF(x)$ rename dummies so both integrals share (firm y, worker x):

$$\int P\,\lambda(1-s)\chi(u_t)f(P)\int_{P' \leq P} \Psi^{\mathrm{UE}}(P')\,dG(P')\,dP = \lambda(1-s)\chi(u_t)\iint x\,\Psi^{\mathrm{UE}}(y)\,\mathbf{1}\{y \leq x\}\,dG(y)\,dF(x),$$
$$\int P\,\lambda(1-s)\chi(u_t)\,\Psi^{\mathrm{UE}}(P)\,g(P)\int_{x \geq P} dF(x)\,dP = \lambda(1-s)\chi(u_t)\iint y\,\Psi^{\mathrm{UE}}(y)\,\mathbf{1}\{x \geq y\}\,dF(x)\,dG(y).$$

Subtracting and using $x\,\mathbf{1}\{y \leq x\} - y\,\mathbf{1}\{x \geq y\} = (x-y)^+$,

$$\lambda(1-s)\chi(u_t)\iint \Psi^{\mathrm{UE}}(y)\,(x-y)^+\,dG(y)\,dF(x) = \lambda(1-s)\chi(u_t)\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(P)\,(p-P)^+\big].$$

EE knowledge diffusion. Write both pieces as double integrals and align arguments of Ψ:

$$\int P\,\lambda s\,g(P)\int_{P' \leq P} \Psi(P', P)\,dG(P')\,dP = \lambda s\iint_{P \geq P'} P\,\Psi(P', P)\,dG(P')\,dG(P),$$
$$\int P\,\lambda s\,g(P)\int_{P' \geq P} \Psi(P, P')\,dG(P')\,dP = \lambda s\iint_{P \geq P'} P'\,\Psi(P', P)\,dG(P')\,dG(P),$$

where in the second line we swapped the variable names $(P, P') \leftrightarrow (P', P)$ to get $\Psi(P', P)$. Subtracting gives

$$\lambda s\iint_{P \geq P'} (P - P')\,\Psi(P', P)\,dG(P')\,dG(P) = \lambda s\,\mathbb{E}\big[\Psi(P_d, P_o)\,(P_o - P_d)^+\big],$$

with (P_o, P_d) i.i.d. from G and $(P_o - P_d)^+ = (P_o - P_d)\,\mathbf{1}\{P_o > P_d\}$. Collecting terms and dividing by $\bar{P}_t$ with $g_t := \frac{d}{dt}\bar{P}_t/\bar{P}_t$

$$\begin{aligned}\frac{d}{dt}\bar{P}_t = {} & \beta\,\mathbb{E}\big[(P' - P)^+\big] + \lambda(1-s)\chi(u_t)\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(P)\,(p-P)^+\big] \\ & + \lambda s\,\mathbb{E}\big[\Psi(P_d, P_o)\,(P_o - P_d)^+\big] + \mu^J\big(\mathbb{E}_\Lambda[P] - \bar{P}_t\big).\end{aligned}$$

such that

$$g_t = \frac{\beta\,\mathbb{E}\big[(P' - P)^+\big]}{\bar{P}_t} + \frac{\lambda(1-s)\chi(u_t)\,\mathbb{E}\big[\Psi^{\mathrm{UE}}(P)\,(p-P)^+\big]}{\bar{P}_t} + \frac{\lambda s\,\mathbb{E}\big[\Psi(P_d, P_o)\,(P_o - P_d)^+\big]}{\bar{P}_t} + \mu^J\left(\frac{\mathbb{E}_\Lambda[P]}{\bar{P}_t} - 1\right).$$

B.6 Model Extension with Product Choice

This appendix extends the baseline single-product environment to allow firms to produce multiple products. The extension facilitates a mapping to empirical settings with multi-product firms (see Bernard et al., 2010).

Environment. Let $\mathcal{I}$ denote the set of products. Each product $i \in \mathcal{I}$ sells at price p_i. A firm j has productivity A_j and chooses product-specific labor $N_{ij} \geq 0$ under a linear technology,

$$y_{ij} = A_j N_{ij}.$$

Product choice occurs after labor mobility is realized, so firm size N_j is fixed at this stage and must satisfy

$$\sum_{i \in \mathcal{I}} N_{ij} = N_j.$$

Activating product i entails a fixed cost F_i; variable labor is paid at the firm-level average wage $\bar{w}_j$. Importantly, our findings on product choice remain robust to different wage-setting protocols, provided that product selection depends on average wages rather than the precise distribution of workers within the firm. This assumption aligns with the notion that firms' production divisions operate with a degree of autonomy from their human resources departments.

Firm problem. Define the per-unit operating margin $m_i(A_j, \bar{w}_j) := A_j p_i - \bar{w}_j$. The firm chooses $\{N_{ij}\}_{i \in \mathcal{I}}$ to maximize

$$\Pi_j \;=\; \max_{\{N_{ij} \geq 0\}} \sum_{i \in \mathcal{I}} \Big(m_i \, N_{ij} - F_i \, \mathbf{1}\{N_{ij} > 0\} \Big) \quad \text{subject to} \quad \sum_{i \in \mathcal{I}} N_{ij} = N_j. \tag{17}$$

Break-even activation and specialization. A product i is (weakly) profitable at scale N iff $m_i > 0$ and $N \geq \tilde{n}_{ij}$, where the break-even labor is

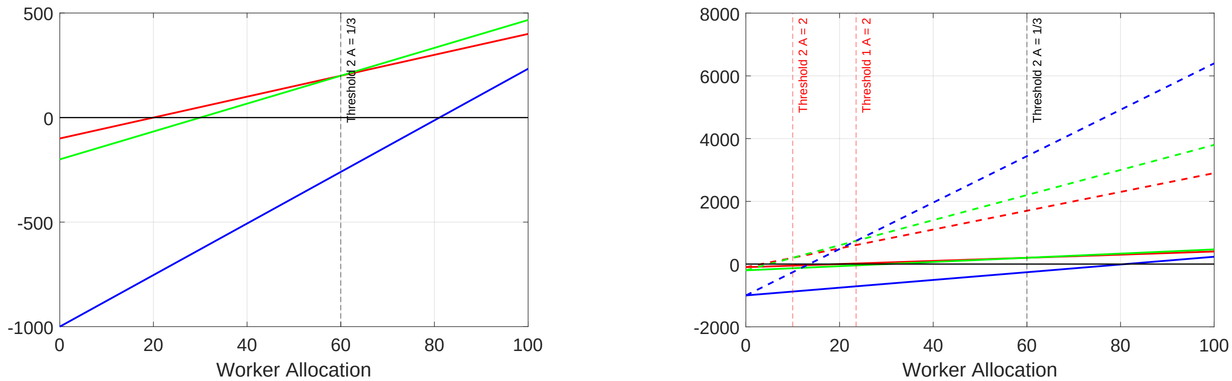
$$\tilde{n}_{ij} \;=\; \frac{F_i}{m_i} \;=\; \frac{F_i}{A_j p_i - \bar{w}_j}.$$

For two products i and k, the firm is indifferent between specializing all labor in i versus k at the labor level N^x_{ikj} that solves

$$m_i N - F_i \;=\; m_k N - F_k \quad \Rightarrow \quad N^x_{ikj} \;=\; \frac{F_k - F_i}{m_k - m_i} \;=\; \frac{F_k - F_i}{A_j(p_k - p_i)},$$

provided $p_k \neq p_i$. If $N^x_{ikj} \notin [0, N_j]$, the firm specializes in the product yielding higher profit at $N = N_j$.

Comparative statics in productivity. An increase in A_j raises each margin m_i and lowers $\tilde{n}_{ij}$, expanding the set of potentially profitable lines. Higher A_j therefore weakly increases the number of products produced and can reallocate labor toward lines with higher implied margins.



(a) Baseline productivity $A_j = \frac{1}{3}$ (b) Higher productivity $A_j = 2$

Figure B.1: Profit functions by product and resulting product choice

Notes: The figures depict per-product profit schedules and the implied allocation across three products. Parameter values: $p_1 = 15$, $p_2 = 20$, $p_3 = 37$, $F_1 = 100$, $F_2 = 200$, $F_3 = 1000$, $N_j = 100$. Panel (a) uses $A_j = 1/3$; panel (b) uses $A_j = 2$.

Appendix C Data Overview

Table C.1: Variables across datasets

Data Set	Period	Variables
Worker data – Swedish Longitudinal Integrated Database for Health Insurance and Labour Market Studies (LISA, Longitudinell Integrationsdatabas för Sjukförsäkrings- och Arbetsmarknadsstudier)	1990-2019	worker id, firm id, age, year of graduation, gender, income, year, occupation, education
Firm data (Företagens Ekonomi)	1997-2019	firm id, firm size, value added, output, capital, year, industry
Customs data (Utrikeshandel med varor)	1997-2019	firm id, export/import flow, value, quantity, year, country
Product data (Industrins Varuproduktion)	1997-2019	firm id, year, product code, value of production, quantity
R&D data (Research and Development survey in private sector)	1997-2019	firm id, year, R&D expenditure

Appendix D Additional Empirical Evidence

D.0.1 Regressions Motivation

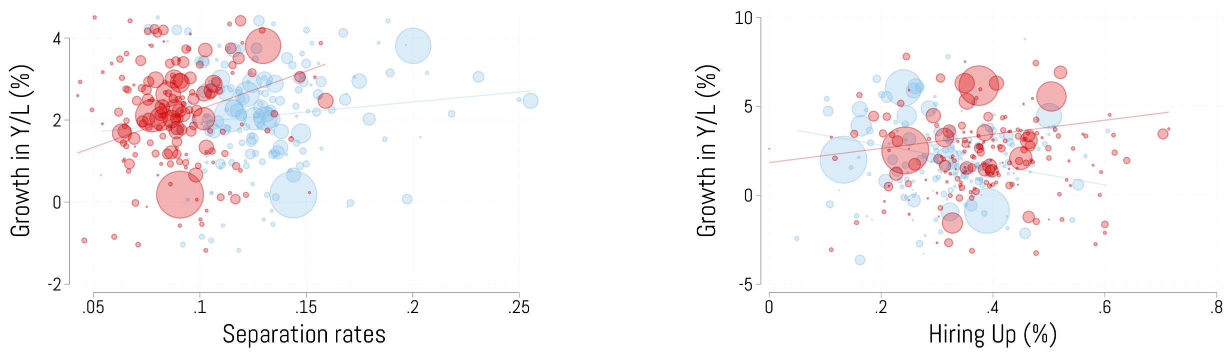


Figure D.1: Productivity growth and mobility (only EE mobility).

Notes: The left column shows binned scatter plots of the separation rate and the growth in labor productivity for occupations with below median wages (blue) and top decile occupations (red).The right hand side shows the share of workers who last worked at higher-productivity firms before entering the current firm and the growth in labor productivity for these two types of workers. Ranking of firms is established in the year prior to mobility based on observed Y/L. A unit of observation is a 4-digit industry. Data for Sweden, 1997–2019.

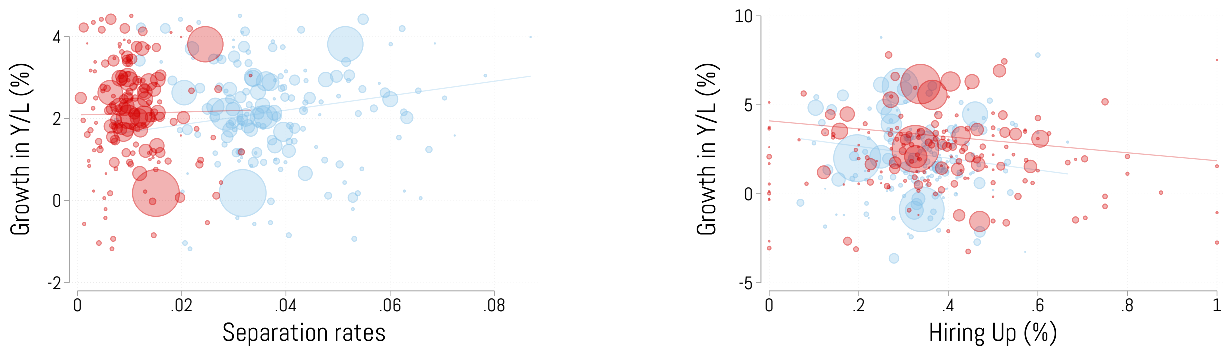


Figure D.2: Productivity growth and mobility (only EU or UE mobility).

Notes: The left column shows binned scatter plots of the separation rate and the growth in labor productivity for occupations with below median wages (blue) and top decile occupations (red).The right hand side shows the share of workers who last worked at higher-productivity firms before entering the current firm and the growth in labor productivity for these two types of workers. Ranking of firms is established in the year prior to mobility based on observed Y/L. A unit of observation is a 4-digit industry. Data for Sweden, 1997–2019.

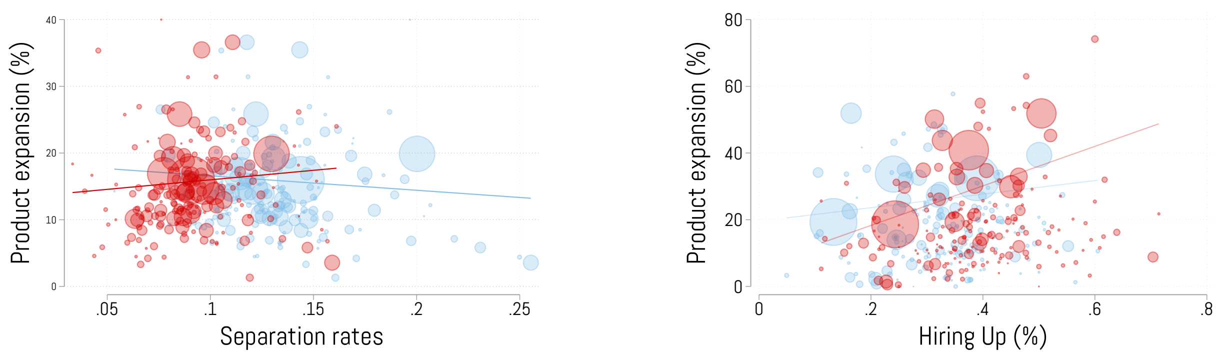


Figure D.3: Product expansion and mobility (only EE mobility).

Notes: The left panel shows binned scatter plots of the separation rate and the likelihood of product expansion for all workers (blue) and top decile occupations as categorized by average earnings (red). The right hand side shows the share of workers who last worked at higher-productivity firms before entering the current firm and the likelihood of product expansion for these two types of workers. Ranking of firms is established in the year prior to mobility based on observed Y/L. A unit of observation is a 4-digit industry and products are denoted at 8-digit. Data for Sweden, 1997–2019.

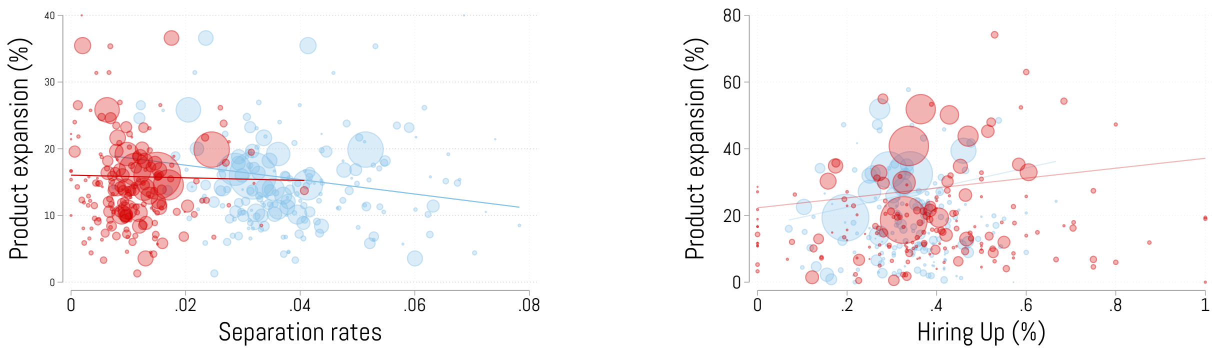


Figure D.4: Product expansion and mobility (only EU or UE mobility).

Notes: The left panel shows binned scatter plots of the separation rate and the likelihood of product expansion for all workers (blue) and top decile occupations as categorized by average earnings (red). The right hand side shows the share of workers who last worked at higher-productivity firms before entering the current firm and the likelihood of product expansion for these two types of workers. Ranking of firms is established in the year prior to mobility based on observed Y/L. A unit of observation is a 4-digit industry and products are denoted at 8-digit. Data for Sweden, 1997–2019.

Table D.1: Regression Results - Labor Productivity

Dep. var.:	Growth in labor productivity					
Controls:	No controls		Y/L		Y/L + volatility	
	Bottom half	Top dec.	Bottom half	Top dec.	Bottom half	Top dec.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Separation rate	0.04*	0.11***	0.10***	0.15***	0.05**	0.06
	(0.02)	(0.04)	(0.02)	(0.03)	(0.03)	(0.04)
Observations	171	171	171	171	170	170
Mean growth	0.02	0.02	0.02	0.02	0.02	0.02
Mean sep. rate	0.16	0.09	0.16	0.09	0.16	0.09
Panel B						
Hiring up	-0.06***	0.03**	-0.00	0.05***	0.03**	0.06***
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
Observations	182	178	182	178	182	178
Mean growth	0.02	0.02	0.02	0.02	0.02	0.02
Mean hiring up	0.33	0.38	0.33	0.38	0.33	0.38
Controls						
Labor productivity			✓	✓	✓	✓
Sector sales volatility					✓	✓
Sector foreign demand volatility					✓	✓

Notes: The table shows regression estimates together with standard errors for the regression specification $\Delta P_{st} = \alpha_0 + \Upsilon_{st}\beta + X\Gamma_t + \epsilon_{st}$ where Y is either the separation rate or the share of hires from higher productivity firms. The unit of observation is a sector. The control vector X contains labor productivity, sales or foreign demand volatility, computed as sector average of firm-specific values.

Table D.2: Regression Results - Product Expansion

Dep. var.:	Product expansion					
Controls:	No controls		Y/L		Y/L + volatility	
	Bottom half	Top dec.	Bottom half	Top dec.	Bottom half	Top dec.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Separation rate	-0.23**	0.15	-0.09	0.30*	-0.46***	-0.12
	(0.10)	(0.17)	(0.11)	(0.16)	(0.12)	(0.20)
Observations	171	171	171	171	170	170
Mean product expansion	0.14	0.14	0.14	0.14	0.14	0.14
Mean sep. rate	0.16	0.09	0.16	0.09	0.16	0.094
Panel B						
Hiring up	0.22***	0.59***	0.25***	0.64***	0.243***	0.66***
	(0.08)	(0.10)	(0.08)	(0.10)	(0.09)	(0.101)
Observations	180	177	180	177	180	177
Mean product expansion	0.17	0.18	0.18	0.18	0.18	0.18
Mean hiring up	0.33	0.38	0.33	0.38	0.33	0.38
Controls						
Labor productivity			✓	✓	✓	✓
Sector sales volatility					✓	✓
Sector foreign demand volatility					✓	✓

Notes: The table shows regression estimates together with standard errors for the regression specification Product Exp. $= \alpha_0 + \mathrm{Y}_{st}\beta + X\Gamma_t + \epsilon_{st}$ where Y is either the separation rate or the share of hires from higher productivity firms. The unit of observation is a sector. The control vector X contains labor productivity, sales or foreign demand volatility, computed as sector average of firm-specific values.

Table D.3: Regression Results - Labor Productivity with Standardized Regressors and R&D

Dep. var.:	Growth in labor productivity					
Controls:	No controls		Y/L		Y/L + volatility	
	Bottom half	Top dec.	Bottom half	Top dec.	Bottom half	Top dec.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Separation rate	0.002**	0.003***	0.004***	0.004***	0.001	0.002*
(standardized)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
R&D activity	0.001	0.001	-0.002***	-0.003***	-0.002***	-0.003***
(standardized)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	171	171	171	171	170	170
Mean growth	0.02	0.02	0.02	0.02	0.02	0.02
Mean sep. rate	0.162	0.09	0.16	0.09	0.16	0.09
Panel B						
Hiring up	-0.005***	0.006***	-0.001	0.006***	0.003*	0.008***
(standardized)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
R&D activity	0.000	0.008***	-0.002	0.004***	0.001	0.005***
(standardized)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)
Observations	182	178	182	178	182	178
Mean growth	0.02	0.02	0.02	0.02	0.02	0.02
Mean hiring up	0.33	0.38	0.33	0.38	0.33	0.38
Controls						
Labor productivity			✓	✓	✓	✓
Sector sales volatility					✓	✓
Sector foreign demand volatility					✓	✓

Notes: The table shows regression estimates together with standard errors for the regression specification $\Delta P_{st} = \alpha_0 + \mathrm{Y}_{st}\beta + X\Gamma_t + \epsilon_{st}$ where Y is either the standardized separation rate or the standardized share of hires from higher productivity firms. We also include the standardized R&D. The unit of observation is a sector. The control vector X contains labor productivity, sales or foreign demand volatility, computed as sector average of firm-specific values.

Table D.4: Regression Results - Product Expansion with Standardized Regressors and R&D

Dep. var.:	Product expansion					
Controls:	No controls		Y/L		Y/L + volatility	
	Bottom half	Top dec.	Bottom half	Top dec.	Bottom half	Top dec.
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A						
Separation rate	-0.000	0.007*	-0.000	0.007*	-0.014***	-0.004
(standardized)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)
R&D activity	0.016***	0.017***	0.017***	0.017***	0.015***	0.017***
(standardized)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)
Observations	171	171	171	171	170	170
Mean product expansion	0.14	0.14	0.14	0.14	0.14	0.14
Mean sep. rate	0.16	0.09	0.16	0.09	0.16	0.094
Panel B						
Hiring up	0.044***	0.083***	0.045***	0.083***	0.051***	0.088***
(standardized)	(0.008)	(0.010)	(0.008)	(0.010)	(0.009)	(0.010)
R&D activity	0.043***	0.070***	0.043***	0.065***	0.048***	0.066***
(standardized)	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)
Observations	180	177	180	177	180	177
Mean product expansion	0.17	0.18	0.18	0.18	0.18	0.18
Mean hiring up	0.33	0.38	0.33	0.38	0.33	0.38
Controls						
Labor productivity			✓	✓	✓	✓
Sector sales volatility					✓	✓
Sector foreign demand volatility					✓	✓

Notes: The table shows regression estimates together with standard errors for the regression specification Product Exp. $= \alpha_0 + \mathrm{Y}_{st}\beta + X\Gamma_t + \epsilon_{st}$ where Y is either the standardized separation rate or the standardized share of hires from higher productivity firms. We also include the standardized R&D. The unit of observation is a sector. The control vector X contains labor productivity, sales or foreign demand volatility, computed as sector average of firm-specific values.

Appendix E Additional Details for Calibration and Solving

E.1 Measurement Details

EE Acceptance and the EE Expectation Identification. Let $h_{\text{out}}(o,t)$ be the per-firm hazard of accepted EE outflow for origin bin o, $S(d \mid o,t)$ the destination share among accepted moves, and $G_t(d)$ the destination-firm share. With firm-level offer arrival at intensity $c_t = \lambda s$,

$$h_{\text{out}}(o,t) = c_t \int \Psi(d,o,t)\, dG_t(d), \tag{18}$$

$$S(d \mid o,t) = \frac{G_t(d)\,\Psi(d,o,t)}{\int G_t(y)\,\Psi(y,o,t)\,dy}. \tag{19}$$

From (18)–(19),

$$\lambda s\,\Psi(d,o,t) = h_{\text{out}}(o,t)\,\frac{S(d \mid o,t)}{G_t(d)}.$$

This delivers a nonparametric estimator of the acceptance kernel from observed hazards, destination shares, and firm shares. On the discrete productivity grid $\mathcal{P} = \{P^1, \ldots, P^K\}$ with firm shares $G_t(k)$, the EE term in (5) has the plug-in form

$$\lambda s\,\mathbb{E}\big[\Psi(P_d, P_o)\,(P_o - P_d)^+\big] = \lambda s \sum_{o=1}^{K}\sum_{d=1}^{K} G_t(o)\,G_t(d)\;\widehat{\Psi}(d,o,t)\,(P^o - P^d)^+, \tag{20}$$

where $\lambda s\widehat{\Psi}(d,o,t)$ is obtained with its empirical counterpart. All objects are measured at the firm-year level; expectations reduce to finite sums over bins, and $x^+ \equiv \max\{x, 0\}$.

Innovation. To capture innovation using R&D data, we proceed as follows. We parameterize the model's innovation arrival as β and estimate firm-level innovation incidence via

$$p_{jt} \;\equiv\; \Pr\!\big(\text{Innov}_{jt} = 1 \,\big|\, z_{jt}\big) \;=\; G\big(z_{jt}'\gamma\big)\,, \tag{21}$$

where $G(\cdot)$ is the logit link, z_{jt} includes R&D outlays, labor productivity, firm size, sector and year fixed effects, and shift–share demand shifters. Let $\widehat{p}_{jt} = G(z_{jt}'\widehat{\gamma})$. We map predicted incidence to the model's arrival rate via the cross-sectional average

$$\widehat{\beta} \;=\; \frac{1}{JT}\sum_{j,t}\widehat{p}_{jt}, \tag{22}$$

Next, define $\mathcal{I}_{jt}$ as an innovation indicator. Focusing on firm–years without EE/UE arrivals, estimate

$$\Delta P_{j,t\to t+\Delta} \;=\; \zeta\,\mathcal{I}_{jt} \;+\; \mathrm{FE}_j \;+\; \mathrm{FE}_{\text{year}} \;+\; \text{demand controls} \;+\; \varepsilon_{jt}, \tag{23}$$

The reduced-form parameter satisfies

$$\zeta = \cdot\mathbb{E}\big[(P' - P)^{+} \,\big|\, \mathcal{I} = 1\big],$$

so the innovation contribution equals $\widehat{\beta} \times \widehat{\zeta}$.

We also capture innovative activity using a residual specification. Let $\dot{\bar{P}}_t/\bar{P}_t$ denote aggregate positive productivity growth and $\widehat{\mathcal{C}}_t^{UE}$, $\widehat{\mathcal{C}}_t^{EE}$, and $\widehat{\mathcal{C}}_t^{J}$ the estimated contributions from UE hiring, EE reallocation, and firm entry in (5). The implied innovation contribution is

$$\widehat{\mathcal{C}}_t^{\text{Innov}} \;=\; \frac{\dot{\bar{P}}_t}{\bar{P}_t} \;-\; \widehat{\mathcal{C}}_t^{UE} \;-\; \widehat{\mathcal{C}}_t^{EE} \;-\; \widehat{\mathcal{C}}_t^{J}. \tag{24}$$

Entry Contribution. Let J_t be the number of active firms at t. Over $[t+1]$, define the firm-count entry rate

$$\mu_t^J \;\equiv\; \frac{\#\{\text{entrants in } [t, t+1]\}}{J_t},$$

Let Λ_t be entrants ; let $\bar{P}_t$ be the incumbent mean at t. Using firm-count weights ,

$$\mathbb{E}_{\Lambda_t}[P] \;\equiv\; \frac{1}{|\Lambda_t|}\sum_{j\in\Lambda_t} P_{j,t},$$

Hence, the contribution deriving from firm entry is

$$\mathcal{C}_t^J \;=\; \mu_t^J\left(\frac{\mathbb{E}_{\Lambda_t}[P]}{\bar{P}_t} - 1\right) \tag{25}$$

UE acceptance and the UE expectation With firm–level arrivals, each firm in bin d faces a UE offer stream at rate $\lambda(1-s)\chi(u_t)$, accepted with probability $\Psi^{UE}(d,t)$ that depends only on the destination. Let $h_{\text{in}}^{UE}(d,t)$ be the per–firm UE accepted inflow hazard into bin d (UE hires per firm). Then

$$\lambda(1-s)\chi(u_t)\,\Psi^{UE}(d,t) \;=\; h^{UE}_{\text{in}}(d,t). \tag{26}$$

The inner expectation becomes

$$\lambda(1-s)\chi(u_t)\mathbb{E}\big[\Psi^{UE}(p)\,(p-P)^{+}\big] = \lambda(1-s)\chi(u_t)\sum_{d}\sum_{\ell} G_t(d)\,F_t(\ell)\,\widehat{\Psi}^{UE}(d,t)\,\big(P^{\ell}-P^{d}\big)^{+}. \tag{27}$$

As in EE, we keep the compound factor $\lambda(1-s)\chi(u_t)\,\widehat{\Psi}^{UE}(d,t) = h^{UE}_{\text{in}}(d,t)$. We approximate F in two ways. First, using its law of motion. (equation 3), we obtain, for the steady-state distribution $f(p)$

$$f(p) = \frac{\delta g(p,t) + \mu^{I}\bar{N}w(p,t)}{u\lambda I^{\downarrow}_{UE}(\infty,t)} = \frac{\delta g(p,t) + \mu^{I}\bar{N}w(p,t)}{E[h^{UE}_{\text{in}}(d,t)]}$$

with

$$I^{\downarrow}_{UE}(P,t) := \int_0^{P} \Psi^{\text{UE}}(P',t)\,dG(P',t).$$

where $\bar{N}$ denotes the average firm size. We assume that $w(p) = g(p)$, such that F is proportional to $G(p)$. Second, we use newly unemployed workers, denoting $\hat{F}$ and approximate $F(l) = \hat{F}(l)$. We present results for both baselines.

E.2 Calibration with Mobility Flows

This appendix describes the calibration and solution approach of the model. We follow Hotz and Miller (1993) and use mobility patterns to infer equilibrium mappings.

Data, objects, and normalizations. We impose stationarity ($\dot{\hat{S}} = \dot{U} = 0$), which allows us to pin down the innovation distribution. Our quantitative results should therefore be interpreted as characterizing growth around this stationary benchmark. In the unemployment–offer law of motion, we set $w(P) = g(P)$ so that the UE offer distribution equals the firm distribution, $f = g$ and $F = G$. The empirical inputs are the firm productivity pdf/cdf (g, G); the inflow distribution of new firms $b(P)$; total EE counts M_{EE} and UE counts M_{UE} over a window of one year; the total number of firms F_{tot}; and the net growth rate of firm counts μ^{J}. Define mobility rates $m_{EE}(o,k)$ and $m_{UE}(k)$. We discretize productivity on a grid $\{P_i\}_{i=1}^{K}$ (bin widths ΔP_i), with masses $\omega_i = g_i\Delta P_i$ normalized so that $\sum_i \omega_i = 1$.

Procedure. We first estimate the innovation distribution on the full productivity grid. We then collapse this grid into a low-productivity group L and a high-productivity group H. Flow ratios and total mover rates then identify the surplus slope and search intensities, allowing us to calibrate the model.

E.2.1 Estimation of Innovation Distribution from Flows and Aggregation

In stationarity, the firm-productivity balance is, for all P,

$$0 = \beta\, v(P)\, G(P) \; - \; \beta\, g(P)\, \bar{V}(P) \; + \; \mu^J \left[b(P) - g(P) \right] \; + \; \mathcal{R}^{EE}_{\phi_E}(P) \; + \; \mathcal{R}^{UE}_{\phi_E}(P).$$

The reallocation terms are

$$\mathcal{R}^{EE}_{\phi_E}(P) = \; \lambda\, s \left[\int_{o<P} g(o)\; \Psi\big(o, k^*(o,P)\big)\; g\big(k^*(o,P)\big)\; \frac{1}{\phi_E}\, \mathrm{d}o \; - \; g(P) \int_{k>P} g(k)\, \Psi(P,k)\, \mathrm{d}k \right],$$

$$\mathcal{R}^{UE}_{\phi_E}(P) = \; \lambda(1-s)\chi(u) \left[\int_{o<P} g(o)\; \Psi^{UE}(o)\; f\big(k^*(o,P)\big)\; \frac{1}{\phi_E}\, \mathrm{d}o \; - \; g(P)\, \Psi^{UE}(P)\, \bar{F}(P) \right].$$

Define $H_{\phi_E}(P)$ by

$$\beta \left[v(P)G(P) - g(P)\bar{V}(P) \right] = H_{\phi_E}(P), \tag{28}$$

with

$$\begin{aligned} H_{\phi_E}(P) = &- \mu^J \left[b(P) - g(P) \right] \\ & - \frac{1}{F_{\text{tot}}} \Bigg\{ \underbrace{\int_{o<P} \frac{1}{\phi_E}\; m_{EE}\big(o,\, k^*(o,P)\big)\, \mathrm{d}o}_{\text{EE inflow to } P} - \underbrace{\int_{k>P} m_{EE}(P,k)\, \mathrm{d}k}_{\text{EE outflow from } P} \\ & + \underbrace{\int_{o<P} \frac{1}{\phi_E}\; m_{UE}(o)\, f\big(k^*(o,P)\big)\, \mathrm{d}o}_{\text{UE inflow to } P} - \underbrace{m_{UE}(P)\, \bar{F}(P)}_{\text{UE outflow from } P} \Bigg\}. \end{aligned}$$

Let $Z(P) := \beta\, \bar{V}(P)$. The linear ODE $G(P)Z'(P) + g(P)Z(P) = -H_{\phi_E}(P)$ implies

$$\beta\, \bar{V}(P) = \frac{1}{G(P)} \int_P^{\infty} H_{\phi_E}(x)\, \mathrm{d}x, \qquad \beta\, v(P) = -Z'(P),$$

using $\lim_{x\to\infty} G(x)Z(x) = 0$ (since $\bar{V}(\infty) = 0$). We then normalize $\int \beta v(P)\, \mathrm{d}P = \beta$.

Data aggregation. We start from a fine grid $\{P_i\}_{i=1}^K$ with firm pdf $g(P)$, entrant pdf $b(P)$, and UE/EE flows $m_{UE}(i)$ and $m_{EE}(i,j)$ across origin–destination bins. We partition the grid into two groups, L and H, of approximately equal firm mass and define aggregated objects

$$g_L = \sum_{i \in L} g_i, \quad g_H = \sum_{i \in H} g_i, \qquad m_{UE,L} = \sum_{i \in L} m_{UE}(i), \quad m_{UE,H} = \sum_{i \in H} m_{UE}(i),$$

$$m_{EE,LH} = \sum_{i \in L} \sum_{j \in H} m_{EE}(i,j), \quad m_{EE,HL} = \sum_{i \in H} \sum_{j \in L} m_{EE}(i,j),$$

and analogous aggregates for entrants and innovation intensities. Let P_L and P_H denote firm–mass–weighted average productivities within L and H, respectively.

E.2.2 Model Calibration using Mobility Patterns

The surplus equation is

$$\begin{aligned} r\hat{S}(P) &= (P - rU) \\ &\quad + \lambda(1-s)\,\chi(u)\,\bar{\Gamma}_u\Big(-\tfrac{\hat{S}(P)}{U}\Big)\int_P^\infty \big(\hat{S}(T_E(P,P')) - \hat{S}(P)\big)\,dF(P') \\ &\quad + \lambda s \int_P^\infty \bar{\Gamma}\Big(1 - \tfrac{\hat{S}(P)}{\hat{S}(P')}\Big)\big(\hat{S}(T_E(P,P')) - \hat{S}(P)\big)\,dG(P') \\ &\quad + \beta \int_P^\infty \big(\hat{S}(P') - \hat{S}(P)\big)\,dV(P'). \end{aligned}$$

Specializing to the low- and high-productivity cells, and using that knowledge flows are only possible from high to low bins, we obtain

$$\begin{aligned} r\hat{S}_L &= P_L - rU \\ &\quad + \phi_E\Big[\alpha_{UE}\,\pi_F^H\,\bar{\Gamma}_u\Big(-\tfrac{\hat{S}_L}{U}\Big) + \alpha_{EE}\,\pi_G^H\,\bar{\Gamma}\Big(1 - \tfrac{\hat{S}_L}{\hat{S}_H}\Big)\Big]\Big(\hat{S}_H - \hat{S}_L\Big) + \beta\,\pi_V^H\,\Big(\hat{S}_H - \hat{S}_L\Big) \\ r\hat{S}_H &= P_H - rU \end{aligned}$$

after defining $\alpha_{UE} = \lambda(1-s)\,\chi(u)$ and $\alpha_{EE} = \lambda s$. Here we use the standard linear approximation on the grid

$$\hat{S}_L(P_E) \;\approx\; (1-\phi_E)\,\hat{S}_L \;+\; \phi_E\,\hat{S}_H.$$

For solving, we re-parametrize the equation with $\hat{S}_L/\hat{S}_H = 1/\rho$ and $\hat{S}_L/U = \tau$

$$r\tau U = P_L - rU \quad + \left[\phi_E\Big(\alpha_{UE}\,\pi_F^H\,\bar{\Gamma}_u(-\tau) + \alpha_{EE}\,\pi_G^H\,\bar{\Gamma}\Big(1-\tfrac{1}{\rho}\Big)\Big) + \beta\,\pi_V^H\right](\rho-1)\tau U$$

$$r\rho\tau U = P_H - rU.$$

We parametrize the mobility shock distributions as

$$\bar{\Gamma}_u = \Lambda(-\tau), \qquad \bar{\Gamma} = \Lambda\Big(\delta_e\big(1-\tfrac{1}{\rho}\big)\Big), \quad \Lambda(x) = \frac{1}{1+e^{-x}}.$$

Together with

$$R_{EE} := \frac{m_{EE,HL}}{m_{EE,LH}} = \frac{\Gamma_{ee,HL}}{\Gamma_{ee,LH}} = \frac{\Lambda\big(\delta_e - (1-1/\rho)\big)}{\Lambda\big(\delta_e - (1-\rho)\big)}$$

$$R_{UE} := \frac{m_{UE,H}}{m_{UE,L}} \cdot \frac{\omega_L}{\omega_H} = \frac{\Gamma_{u,H}}{\Gamma_{u,L}} = \frac{\Lambda(-\tau\rho)}{\Lambda(-\tau)},$$

we obtain a system of four equations in four unknowns τ, ρ, δ_e, U. Given $(\rho, \tau, \delta_e, U)$, acceptance probabilities are known. Arrival rates follow from level identities:

$$\alpha_{UE} = \frac{(m_{UE,L} + m_{UE,H})/F_{\text{tot}}}{\omega_L\,\Gamma_{u,L} + \omega_H\,\Gamma_{u,H}}, \qquad \alpha_{EE} = \frac{(m_{EE,LH} + m_{EE,HL})/F_{\text{tot}}}{\omega_L\,\omega_H\,[\,\Gamma_{ee,LH} + \Gamma_{ee,HL}\,]}.$$